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Résumé de l'article

The existing body of literature underscores the crucial role of technology, driven by both innovation and imitation, in fostering economic growth. Human capital emerges as a key factor influencing technology adoption and innovation. We consider a R&D-based growth model to analyze how improvement in schooling quality impacts technical progress (via the twin channels of imitation and innovation) and therefore, long- run economic growth of an economy by working through the influence of fertility rates and education decisions at household level. The results indicate that improvement in schooling quality triggers a child quantity-quality trade-off at the household level when quality of schooling exceeds an endogenously determined threshold. At household level, parents invest more in the education of their children and have lesser number of children. This micro-level trade-off has two opposing effects on aggregate human capital accumulation at the macroeconomy wide level. A higher investment in education of a child stimulates the accumulation of human capital, which fosters technical progress, but the simultaneous decline in fertility rate reduces total factor productivity growth by contracting the stock of human capital. The former effect prevails over the latter only when quality of schooling is higher than the threshold and therefore, economic growth is driven by rate of aggregate human capital accumulation under both innovation and imitation regimes. However, when the quality of schooling is lower than this threshold, parents do not invest in education and focus on maximizing fertility. Therefore, the economy grows at the rate of population growth at the macro level under the two regimes. Also, it is advantageous for an economy to innovate upon the local technology frontier instead of imitating from the world technology frontier if the rate of human capital accumulation is higher than the growth rate of world technology frontier in the presence of constant or diminishing returns to R&D sector.

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Quality of Schooling, Fertility and Economic Growth

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The existing body of literature underscores the crucial role of technology, driven by both innovation and imitation, in fostering economic growth. Human capital emerges as a key factor influencing technology adoption and innovation.We consider a R&D-based growth model to analyze how improvement in schooling quality impacts technical progress (via the twin channels of imitation and innovation) and therefore, long- run economic growth of an economy by working through the influence of fertility rates and education decisions at household level. The results indicate that improvement in schooling quality triggers a child quantity-quality trade-off at the household level when quality of schooling exceeds an endogenously determined threshold. At household level, parents invest more in the education of their children and have lesser number of children. This micro-level trade-off has two opposing effects on aggregate human capital accumulation at the macroeconomy wide level. A higher investment in education of a child stimulates the accumulation of human capital, which fosters technical progress, but the simultaneous decline in fertility rate reduces total factor productivity growth by contracting the stock of human capital. The former effect prevails over the latter only when quality of schooling is higher than the threshold and therefore, economic growth is driven by rate of aggregate human capital accumulation under both innovation and imitation regimes. However, when the quality of schooling is lower than this threshold, parents do not invest in the education and focus on maximizing fertility. Therefore, the economy grows at the rate of population growth at the macro level under the two regimes. Also, it is advantageous for an economy to innovate upon the local technology frontier instead of imitating from the world technology frontier if the rate of human capital accumulation is higher than the growth rate of world technology frontier in the presence of constant or diminishing returns to R&D sector.

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1 Introduction

According to Barro & Lee (2013) estimates, the share of population without any formal schooling in developing countries has declined from 54.6 percent in 1960 to approximately 17.4 percent in 2010. However, merely expanding access to education does not ensure that children actually learn in schools. The learning outcomes in schools closely hinge upon the quality of schooling, which has been given inadequate attention in the development policy paradigms of most developing countries until now. But more recently, development policies of most countries are making a shift towards improving learning quality in schools than merely expanding access to education.

This policy paradigm shift in education policy is also reflected in the post-2015 development agenda. Imparting quality education features as the fourth Sustainable Development Goal set by the United Nations. This shift is motivated by two factors. First, there is growing empirical evidence that quality of schooling matters more for economic growth. Several studies have found that human capital quality has a significant positive impact on growth (Hanushek & Kimko 2000, Hanushek & Woessmann 2012, Ciccone & Papaioannou 2009, Islam et al. 2014, Neycheva 2016, Altinok & Aydemir 2017, Campbell & Üngör 2020). Second, poor quality of schooling remains a dismal reality in developing countries. According to the Annual Status of Education Report (ASER) (2019) survey titled 'Early Years', at least 25 percent of Indian school children in the four-eight age group do not have age-appropriate cognitive and numeracy skills, making for a massive learning deficit at an early age. Similarly, Glewwe et al. (2010) report that teachers from rural schools in Kenya were absent 20 percent of the time; while, in Zambia and Pakistan, teachers were absent, respectively, 18 percent and 10 percent of the time (Das et al. 2004, Reimers 1993).

In addition to these observations about quality of schooling, certain demographic changes have been observed in the world since past few decades. The total fertility rate has declined markedly in many regions of the world over the last few decades.[1] The average fertility has declined in sub-Saharan Africa from 6.3 births per woman in 1990 to 4.6 in 2019. Other regions have also witnessed a fertility decline over the same period - Northern Africa and Western Asia (from 4.4 to 2.9), Central and Southern Asia (4.3 to 2.4), Eastern and South-Eastern Asia (2.5 to 1.8), Latin American and the Carribean (3.3 to 2.0) and the Oceania (4.5 to 3.4)[2](United Nations, Department of Economic and Social Affairs, Population Division 2019). Around 40 percent of the world's population lives in intermediate-fertility countries and close to 50 percent

[1]Total fertility rate is defined as the average number of children born to women over a lifetime.

[2]The region of Oceania excludes New Zealand and Australia in the UN Report.

of the global population lives in low-fertility countries.[3] It is expected that slightly less than 30 percent of the world population will live in intermediate-fertility countries and 70 percent of the total population will live in low-fertility countries in 2050.

Human capital is a direct factor of production, which is positively related to output growth just like other factors, such as physical capital and labor (Lucas Jr 1988, Rebelo 1991, Mankiw et al. 1992). Additionally, human capital facilitates the adoption and development of technology (sometimes differentiated by imitation and innovation activity as two distinct routes for technological progress) (Nelson & Phelps 1966, Benhabib & Spiegel 1994). Using a horizontal R&D-based growth model á la Romer (1990) with endogenous human capital supply, Boikos et al. (2022) model leisure externalities in R&D activity and examine their impact on innovation rate when interacted with technological spillovers. They show that ceteris paribus, leisure has a positive influence on innovation activity if there are weak intertemporal spillover effects arising from existing ideas. However, if there exists strong intertemporal spillover effects, then, leisure has a negative influence on innovation activity. As it is evident from this endogenous growth literature, human capital is a major determinant of economic growth. A decline in population implies a decline in human capital which can lead to a decline in economic growth as human capital is the driving force for R&D activities. Thus, declining fertility rate can strangle economic growth.

However, empirical literature finds that population growth and economic growth are negatively correlated (Kelley & Schmidt 1994, Brander & Dowrick 1994, Li & Zhang 2007, Herzer et al. 2012, Prettner et al. 2013). Boikos et al. (2013) investigates the relationship between the fertility rate and per capita human capital investment. Using a theoretical framework, they show that birth rate has a monotonically negative impact on economic growth rate when birth rate is exogenously given and it is assumed that its impact on per capita human capital investment is linear and monotonically negative. In the absence of any specific assumption regarding impact on per capita human capital investment and after endogenizing birth rate, they show that total impact of population growth on economic growth is non-monotonic which conforms with the possibility that impact of population growth on economic growth may differ (in sign and magnitude) across countries characterized by different birth rates.

There exists a strand of theoretical literature linking R&D based growth with endogenous fertility and education decisions (Strulik 2005, Strulik et al. 2013, Hashimoto & Tabata 2016, Bucci et al. 2020) that provides an explanation of why empirical literature finds no supportive evidence of the pessimistic prediction that declining fertility can strangle economic growth. The child quantity-quality trade-off at the micro level is posited as one of the plausible explanations.

[3]Intermediate-fertility countries are countries (such as India, Indonesia, Pakistan, the Philippines and Egypt) where women have an average lifetime fertility that ranges between 2.1 and 4 live births. Low-fertility countries are those countries where fertility is below 2.1 live births per woman. It includes almost all of Europe, Northern America, Australia and New Zealand.

This micro level trade-off ensures that declining fertility is accompanied by higher human capital endowment per person in terms of education and health. As a result, although, declining fertility reduces workforce size but, also, leads to higher human capital endowment per person. This higher human capital accumulation averts the negative economic impact of declining fertility. Given that human capital is the driving force for R&D, this entails a higher R&D output and higher R&D-based growth.

This strand of literature uses discrete-time overlapping generations framework to analyze the effect of child quantity-quality trade-off on economic growth. Rise in life expectancy of parents (Hashimoto & Tabata 2016),increase in health investment of children (Baldanzi et al. 2021), rising wages (Strulik et al. 2013) are some of the common mechanisms used to explain child quantity-quality trade-off at the household level and its consequent impact on R&D-based economic growth.[4] In a slight deviation from the usual mechanism of child quantity-quality trade-off adopted by most of the studies in this area, Cervellati et al. (2023) utilize a two-sector framework to show how complementarities between human and physical capital yield endogenous dynamics of population, physical capital, and human capital. Our work is related to this strand of literature linking R&D based growth with endogenous fertility and education decisions and attempts at showing quality of schooling can be another mechanism that can trigger child quantity-quality trade-off at the household level.

There exists another strand of theoretical literature that analyzes the linkages between quality of schooling and economic growth. Many existing studies (Tamura 2001, Gilpin & Kaganovich 2012) on quality of schooling and economic growth focus on explaining how determinants of quality of schooling, such as teacher-student ratio and teacher quality together, impact the learning process, and the consequent human capital formation and, therefore, economic growth. However, most of these studies assume exogenously determined population growth and also do not consider technical progress in their models. Consequently, these studies are unable to analyze the impact of schooling quality and the resulting demographic change on R&D activities, which are major determinants of technological development in the real world. We improve upon these papers by endogenizing both - population growth and technical change.

The concept that the technology diffusion plays a crucial role in economic growth has a rich historical background. Nelson & Phelps (1966) was one of the first studies to highlight the

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[4] In general, this child quantity-quality trade-off between fertility and education is a crucial component of unified growth theory, which models the transition from Malthusian state to sustained growth of an economy. The genesis of this strand of literature can be traced back to the seminal work of Becker (1960). Galor & Weil (2000) postulate that a higher rate of technical progress triggers a child quantity-quality trade-off which induces a demographic transition in which fertility rates decline and investments in human capital of children increase. This, eventually, paves way to the period of sustained economic growth of an economy. Besides technical progress, declining child mortality (Soares 2005), rise in life expectancy of parents (Boucekkine et al. 2002, 2003, Kalemli-Ozcan 2002, 2003), and decline in gender wage gap (Galor & Weil 1996) are other mechanisms to explain child quantity-quality trade-off at the household level and the consequent long-run development from stagnation to modern growth of economies.

role of human capital in facilitating technology diffusion. Jovanovic & Rob (1989) formulate an aggregate model featuring diverse agents, where interactions generated both novel ideas and the replication of ideas—a process of diffusion from initially more productive agents to less productive ones. Jovanovic & MacDonald (1994) delved into innovation and diffusion within a competitive industry, exploring how firm-level incentives were influenced by the distribution of technologies among their competitors. Kortum (1997) introduced the concept of a technology frontier to describe the knowledge state within a society. Lucas Jr (2009, 2015) developed models where this frontier evolved through interactions among agents, leading to the transfer of knowledge. Lucas Jr & Moll (2014) expanded on these models, allowing agents to allocate their time between production and acquiring knowledge. Perla & Tonetti (2014) show that growth is generated as a positive externality from risk-taking by less productive firms imitating more productive firms, leading to technology diffusion and sustained growth. Benhabib et al. (2014) develop frameworks in which agents optimally choose the amount to invest in improving growth through innovation as well as through technology diffusion. Benhabib et al. (2021) extend the analysis further and show how innovation and technology diffusion interact to endogenously determine the shape of the productivity distribution and generate aggregate growth. Stokey (2021) look at technology-skill complementarity and explain how the interplay between skill acquisition and technology drives economic growth in a stylized economy.

The empirical literature on technology diffusion employs various methodologies, examining geographic diffusion within a country, diffusion among firms within an industry, and cross-country diffusion respectively. Griliches (1957) is an early study on hybrid corn adoption in U.S. that finds hybrid adoption across geographic regions is well explained by their relative profitability across regions. Foster & Rosenzweig (1996) and Foster & Rosenzweig (2010) looks at introduction of high-yielding varieties in India and across multiple countries respectively and find that schooling played a crucial role in explaining variations. Along similar lines, Manuelli & Seshadri (2014) studied the gradual diffusion of tractors in U.S. agriculture. Comin & Hobijn (2004) explore the cross-country diffusion of specific technologies over two centuries, analyzing 23 industrial economies and Comin & Hobijn (2010) expand their study to 166 countries encompassing the entire spectrum of income levels.In the former investigation, they identify human capital as a crucial factor influencing the speed of adoption. Building upon this, Comin & Mestieri (2018) delve into adoption lags and the intensity of technology use. The findings suggest that variations in adoption lags played a significant role in the cross-country income disparities during the nineteenth century, while distinctions in intensity of use further contributed to divergence in the twentieth century.

As it is evident from the literature, human capital plays a pivotal role in facilitating technology diffusion. In an influential empirical study, Krueger & Lindahl (2001) observe that human capital enhances growth only for the countries with the lowest level of education. That is, education matters only for catching up but not for innovation at the frontier. In an attempt to resolve this Krueger-Lindahl puzzle, Vandenbussche et al. (2006) argue that human capital does not affect innovation and imitation uniformly. They develop an endogenous growth model, where innovation makes relatively more intensive use of skilled labor and imitative activities make relatively intensive use of unskilled labor and show that skilled labor has a higher growth-enhancing effect closer to the technological frontier. Using a panel dataset covering 19 OECD countries for period 1960-2000, they find evidence in support of their theoretical findings.[5] Ang et al. (2011) empirically investigate the predictions of the theoretical model of Vandenbussche et al. (2006) for developing countries. Their results show that the growth enhancing effects of tertiary education attainment or skilled human capital increase when high and medium income countries move closer to the technology frontier. Human capital is not contributing to growth in low income countries, suggesting that they neither innovate nor imitate.

Thus, it can be concluded from the extant literature that technology (via the twin channels of innovation and imitation) is a pivotal driver of economic growth and human capital plays a fundamental role in technology adoption and innovation in the process of technical progress. The endogenous fertility and education decisions at the household level can influence human capital accumulation at the aggregate level, which in turn, can affect the economic growth via its impact on technical progress. However, impact on economic growth can differ depending upon whether innovation or technology adoption is driving technical progress.

We try to integrate these different strands of literature by building an overlapping generations version of an R&D-based growth model á la Diamond (1965) and Jones (1995) to examine the impact of a child quantity-quality trade-off triggered by improvement in quality of schooling on technical progress and economic growth. We focus on characterizing two cases of high and low quality of schooling for an economy and examine the corresponding drivers of economic growth - rate of human capital accumulation and population growth in these two cases under two distinct regimes of technological improvement - innovation and imitation. Under the innovation regime, technological improvements occur by innovating upon local technology frontier, whereas technological progress occurs by imitating existing foreign technologies under imitation regime. We derive the condition under which it will be advantageous for an economy to innovate on local technology frontier. In this respect as well, this work is an improvement over existing research in this area.

We find that the quality of schooling triggers a child quantity-quality trade-off at the micro

[5] They show that human capital affects the rate of technical progress via a level effect and a composition effect. Holding the composition of human capital constant, an increase in the stock of human capital is always growth-enhancing. However, holding its level constant, the growth-enhancing properties of human capital depend on both its composition and the distance to the technological frontier. The growth-enhancing impact of skilled labor increases with a country's proximity to the world technology frontier, where proximity is measured by the ratio between the total factor productivity in the country and the corresponding variable for a frontier economy such as US. Conversely, the growth-enhancing impact of unskilled labor decreases with the proximity to the world technology frontier.

level when quality of schooling surpasses an endogenously determined threshold under both the technology regimes. When quality of schooling surpasses the threshold, parents invest in education of their children and bear fewer number of children. This micro-level trade-off has two opposing effects on aggregate human capital accumulation at the macro level. A higher investment in the education of a child stimulates the accumulation of human capital, which fosters technical progress, but the simultaneous decline in fertility rate reduces the total factor productivity growth and economic growth by contracting the pool of available researchers. The first effect prevails over latter only when quality of schooling is higher than the threshold. When quality of schooling is less than the threshold, parents do not educate their children and focus on maximizing fertility. In such a scenario, economic growth is solely driven by population growth under both innovation and imitation regimes.Distance from technology frontier is another driver of growth under imitation regime.

Our results show that it is advantageous for an economy to innovate upon the local technology frontier instead of imitating from the world technology frontier if the rate of human capital accumulation is higher than the growth rate of world technology frontier in the presence of constant or diminishing returns to R&D sector. Furthermore, a mere surpassing of the threshold level of quality schooling is not sufficient enough for an economy to experience a higher economic growth rate as compared to an economy with quality of schooling lower than the threshold level. Under the two technology regimes, quality of schooling should be high enough such that it leads to high enough investments in education of children, entailing that the growth-stimulating effect dominates the growth-impeding effect of quality of schooling.

The rest of the paper is organized as follows. Section 2 discusses the basic structure of the model. Sections 3 and 4 discuss the market clearing condition and the key analytical results for a decentralized economy, which provide the key propositions of this study. Section 5 concludes.

2 The Model and Equilibrium Solutions

2.1 The Economic Environment

We consider a model economy populated by overlapping generations of people, each of whom lives for two periods: adulthood and old age. Time is discrete and spans from 0 to ∞. During childhood, which is not modeled explicitly, individuals are reared and educated by their parents. All the decisions are made at the beginning of adulthood. Adults are identical in all aspects.They inelastically supply their skills in the labor market. Adults care about the consumption of a homogeneous final good, number and human capital level of their children. During old age, individuals consume their savings plus interest earned on these. Abstracting from gender differences, each household has a single parent. For avoiding the indivisibility problem, we assume that children are borne in continuous number. All individuals survive up to adulthood. The education of current period's children (though childhood is not modeled explicitly) determines human capital endowment of next period's adult generation. Akin to Castelló-Climent & Hidalgo-Cabrillana (2012), human capital accumulation function depends on an exogenously given quality of education, parental investment in education and human capital of parent. Parental investment in education is a fraction of income spent on education of each child.

The production structure of the economy closely follows Romer (1990) and Jones (1995). The economy consists of three sectors: final goods, intermediate goods and R&D. Perfect competition prevails in the final goods and R&D sectors whereas monopoly prevails in the intermediate goods sector. The intermediate goods are horizontally differentiated, each produced with the help of a blueprint design developed in the R&D sector. The entire range of intermediate goods constitutes as an input in the final goods sector.

2.2 Individuals

Individuals derive utility from $c_{1,t}$, their own consumption of the final good during adulthood; $c_{2,t+1}$, their own consumption during old age; n_t, number of children borne and h_{t+1}, human capital per child. Parents' motivation to invest in human capital of children by spending on children's education is driven by a 'warm glow' of giving (Andreoni 1989) or preference for having 'higher-quality' children (Becker 1960). The lifetime expected utility of individuals in generation t is given by:

$$u_t = \log c_{1,t} + \beta_1 \log c_{2,t+1} + \beta_2 \log(h_{t+1} n_t), \tag{1}$$

where positive weights β_1 and β_2 measure the weights on future consumption, $c_{2,t+1}$, child quantity, n_t and quality, h_{t+1} relative to current consumption, c_t, in the utility function. Alternatively, following De la Croix & Doepke (2004), β_2 can be interpreted as an "altruism" factor.

An individual's embodied human capital is denoted by h_t and the wage per unit of human capital is w_t. Young adults spend their income on current consumption, savings for old-age consumption and child's education expenditure. Rearing a child necessarily takes fraction $\tau \in (0,1)$ of an adult's time, which is given exogenously. Accordingly, the budget constraints for the adults and old individuals are given by:

$$w_t h_t (1 - \tau n_t) = c_{1,t} + s_t + e_t (w_t h_t) n_t; \tag{2}$$

$$c_{2,t+1} = (1 + r_{t+1}) s_t, \tag{3}$$

where e_t is the fraction of income per child spent on education, s_t is savings and r_{t+1} is interest rate. Non-negativity constraints apply to all the variables.

The human capital of children, h_{t+1}, depends on human capital of parents, h_t, parental investment in education per child, e_t, and quality of education system, θ, which is exogenously given.

$$h_{t+1} = (\mu + \theta e_t)^{\epsilon} h_t. \tag{4}$$

The parameters satisfy $\mu \geq 1$ and $\epsilon \in (0,1)$. ϵ measures the returns to education. μ denotes the inter-generational human capital spillovers, that is, basically skills learnt by children by observing and imitating parents. The parametric restriction of $\mu \geq 1$ ensures that the growth rate of per capita human capital does not become negative when parents do not invest in education. It ensures that children will acquire knowledge and skills atleast equivalent to their parents when parents do not educate their children.

The assumption that quality of schooling is an argument in human capital accumulation function is consistent with a number of other studies (Hanushek et al. 2008, Castelló-Climent & Hidalgo-Cabrillana 2012). Parental human capital, h_t, as an input in human capital accumulation technology represents inter-generational transfers of human capital, which is a common assumption in the literature (De la Croix & Doepke 2004, Tamura 2001, Kalemli-Ozcan 2002, 2003).

Individuals maximize utility in eq. (1) with respect to the constraints, eqs. (2) to (4) using control variables $c_{1,t}$, s_t, n_t and e_t. The solution to individuals' decision problem can either be interior, or at a corner where the individuals choose zero education. The first-order conditions yield the solutions in eqs. (5) to (8), for consumption and savings, irrespective of whether education acquisition level is in the interior or at the corner:[6]

$$c_{1,t} = \frac{w_t h_t}{1 + \beta_1 + \beta_2}; \tag{5}$$

$$s_t = \frac{\beta_1 w_t h_t}{1 + \beta_1 + \beta_2}. \tag{6}$$

For child quantity and quality, there exists a threshold level of quality of schooling. If quality of schooling falls below the threshold, adults do not spend on child quality (or education) and maximize child quantity or the number of children borne. This constitutes the corner solution. In particular, following results are derived from the first-order conditions:

$$e_t = \begin{cases} 0, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon}; \\ \frac{\tau\theta\epsilon - \mu}{\theta(1-\epsilon)}, & \text{otherwise,} \end{cases} \tag{7}$$

$$n_t = \begin{cases} \frac{\beta_2 \epsilon \theta}{(1 + \beta_1 + \beta_2)\mu}, & \text{if } \theta < \frac{\mu}{\tau\epsilon}; \\ \frac{\beta_2}{(1 + \beta_1 + \beta_2)\tau}, & \text{if } \theta = \frac{\mu}{\tau\epsilon}; \\ \frac{\beta_2 \theta(1-\epsilon)}{(1 + \beta_1 + \beta_2)(\tau\theta - \mu)}, & \text{otherwise.} \end{cases} \tag{8}$$

Inserting eq. (7) in eq. (4), we get an equation of motion for human capital as:

$$h_{t+1} = \begin{cases} \mu^{\epsilon} h_t, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon}; \\ \left[\frac{\epsilon(\tau\theta - \mu)}{(1-\epsilon)}\right]^{\epsilon} h_t, & \text{otherwise.} \end{cases} \tag{9}$$

[6]Detailed mathematical derivations are provided in Appendix A.

Below the threshold in eq. (9), quality of schooling is not an argument in the human capital production function. Without education expenditure, human capital of the next generation consists of basic skills only. From eqs. (5) to (8), irrespective of whether quality of schooling exceeds the threshold or not, savings and consumption are increasing in $w_t h_t$, and there is no direct effect of income on fertility because a positive income effect of an increase in wages on fertility is balanced by a negative substitution effect on fertility. The quality of schooling has a direct bearing on child quantity and quality. The following lemma says how quality of schooling influences fertility behavior.

Lemma 1 *When quality of schooling is high enough to surpass the threshold, a marginal improvement in the quality of schooling triggers a child quantity-quality trade-off such that adults bear lesser number of children and invest more in the education per child in response to improvement in quality of schooling (that is, $\frac{\partial n_t}{\partial \theta} < 0$ & $\frac{\partial e_t}{\partial \theta} > 0$ when $\theta > \frac{\mu}{\tau\epsilon}$). However, when quality of schooling is lower than the threshold, it has no effect on child quality as adults do not invest in child's education and focus instead on maximizing child quantity (that is, $\frac{\partial n_t}{\partial \theta} > 0$ & $\frac{\partial e_t}{\partial \theta} = 0$ when $\theta < \frac{\mu}{\tau\epsilon}$). Furthermore, child quantity does not depend on quality of schooling when quality of schooling equals the threshold (that is, $\frac{\partial n_t}{\partial \theta} = 0$ & $\frac{\partial e_t}{\partial \theta} = 0$ when $\theta = \frac{\mu}{\tau\epsilon}$)*

Proof. By investigating the corner solutions in eqs. (7) and (8), it can be immediately seen that quality of schooling entails no child quantity-quality trade-off if quality of schooling falls below the threshold. Adults do not spend on education and maximize fertility when quality of schooling is strictly less than the threshold. The quality of schooling has no effect on fertility when quality of schooling equals the threshold. To analyze the effect when quality of schooling is above the threshold, we take the derivatives of the interior solution of e_t and n_t with respect to θ in eqs. (7) and (8). That is,

$$\frac{\partial n_t}{\partial \theta} = \frac{-\mu\beta_2(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2} < 0; \tag{10}$$

$$\frac{\partial e_t}{\partial \theta} = \frac{\mu}{(1-\epsilon)\theta^2} > 0. \tag{11}$$

When quality of schooling is strictly less than the threshold, the derivatives of the corner solution of e_t and n_t with respect to θ in eqs. (7) and (8) yield:

$$\frac{\partial n_t}{\partial \theta} = \frac{\beta_2\epsilon}{(1+\beta_1+\beta_2)\mu} > 0;$$

$$\frac{\partial e_t}{\partial \theta} = 0.$$

■

Thus, from eqs. (10) and (11) it can be inferred that fertility changes are directly triggered by quality of schooling. Intuitively, any improvement in quality of schooling over and above the threshold makes learning in schools more effective and, therefore, increases marginal returns to investment in human capital. Consequently, a parent reduces fertility, $\frac{\partial n_t}{\partial \theta} < 0$, and spends more on education per child, $\frac{\partial e_t}{\partial \theta} > 0$.
Thus, quality of schooling can be perceived as another plausible mechanism for triggering child quantity-quality trade-off. These theoretical results are in line with recent empirical findings. For example, Hanushek et al. (2008) find that a lower quality of schooling leads to higher dropout rates in Egyptian primary schools. A cross-country analysis by Castelló-Climent & Hidalgo-Cabrillana (2012) reveals that quality of education has a positive effect on enrollment rates in secondary schooling only when quality of schooling is sufficiently high.

Lemma 2 *An increase in returns to education, ϵ, leads to a child quantity-quality trade-off wherein parents educate their children and bear lesser number of children when quality of schooling surpasses the threshold (that is, $\dfrac{\partial e_t}{\partial \epsilon} > 0$ & $\dfrac{\partial n_t}{\partial \epsilon} < 0$ when $\theta > \dfrac{\mu}{\tau\epsilon}$). However, when quality of schooling is less than the threshold, returns to education have no effect on education of children and parents maximize child fertility (that is, $\dfrac{\partial e_t}{\partial \epsilon} = 0$ & $\dfrac{\partial n_t}{\partial \epsilon} > 0$ when $\theta < \dfrac{\mu}{\tau\epsilon}$) and child quantity does not depend on returns to education when quality of schooling equals the threshold (that is, $\dfrac{\partial n_t}{\partial \epsilon} = 0$ & $\dfrac{\partial e_t}{\partial \epsilon} = 0$ when $\theta = \dfrac{\mu}{\tau\epsilon}$) .*

Proof. Taking the derivatives of the interior solution of e_t and n_t with respect to ϵ in eqs. (7) and (8), one gets that:

$$\frac{\partial n_t}{\partial \epsilon} = \frac{-\beta_2\theta}{(1+\beta_1+\beta_2)(\tau\theta-\mu)} < 0;$$

$$\frac{\partial e_t}{\partial \epsilon} = \frac{\tau\theta-\mu}{\theta(1-\epsilon)^2} > 0.$$

When quality of schooling is strictly less than the threshold, the derivatives of the corner solution of e_t and n_t with respect to ϵ in eqs. (7) and (8) yield:

$$\frac{\partial n_t}{\partial \epsilon} = \frac{\beta_2\theta}{(1+\beta_1+\beta_2)\mu} > 0;$$

$$\frac{\partial e_t}{\partial \epsilon} = 0.$$

Also, the returns to education has no effect on fertility when quality of schooling equals the threshold i.e. $\dfrac{\partial n_t}{\partial \epsilon} = \dfrac{\partial e_t}{\partial \epsilon} = 0$ ■

This implies that returns to education are yet another factor that can trigger a child quantity-quality trade-off. A higher return to education implies that education makes human capital more productive. Therefore, parents invest in the education of their children and decide to have lesser number of children. However, when quality of schooling is less than the threshold, parents do not invest in education of children and, therefore, returns to schooling has no effect on child quality and instead, child quantity is maximized when schooling quality is strictly less than the threshold. Both Lemmas 1 and 2 will be used later in our analysis.

In what follows, the production side of the economy is described and solved for the equilibria.

2.3 Final Goods Sector

The final homogeneous good, Y_t, is produced and sold in a competitive market. Firms produce the final good, Y_t, using human capital, H_t^Y, and a range of horizontally differentiated intermediate inputs, x_{it}. The production function for firms is defined as:

$$Y_t = (H_t^Y)^{1-\alpha} \sum_{i=1}^{A_t} x_{it}^{\alpha}, \qquad 0 < \alpha < 1. \tag{12}$$

The parameter, α, is the capital share in final good's production. The price of final good, P_Y, has been normalized to 1. In each period, t, the final good's producers solve the following profit maximization problem with respect to their choice of the range of intermediate inputs and human capital, H_Y:

$$Max_{x_{it}, H_t^Y} \;\; \pi_{t,Y} = (H_t^Y)^{1-\alpha} \sum_{i=1}^{A_t} x_{it}^{\alpha} - w_Y H_t^Y - \sum_{i=1}^{A_t} p_{it} x_{it},$$

where p_{it} is the unit price of ith intermediate input and w_Y is the wage rate prevailing in final good sector. The first-order conditions imply that:

$$p_{it} = \alpha (H_t^Y)^{1-\alpha} x_{it}^{\alpha-1}; \tag{13}$$

$$w_Y = \frac{(1-\alpha)(H_t^Y)^{1-\alpha} \sum_{i=1}^{A_t} x_{it}^{\alpha}}{H_t^Y} = \frac{(1-\alpha)Y_t}{H_t^Y}. \tag{14}$$

Eq. (13) yields the demand for each intermediate input as:

$$x_{it} = \left[\frac{\alpha}{p_{it}}\right]^{\frac{1}{1-\alpha}} H_t^Y. \tag{15}$$

An analysis of the intermediate goods sector ensues.

2.4 Intermediate Goods Sector

Each intermediate good i is produced by monopolist producer who holds the blueprint for the intermediate good i. Any intermediate input producing firm has to acquire new blueprint from R&D sector. Firms issue shares to households to raise funds for buying blueprints. Following

Hashimoto & Tabata (2016) and Futagami & Konishi (2019), it is assumed that each intermediate good uses one unit of human capital in a one-to-one production technology, or $x_{it} = H_{it}$. Thus, the amount of intermediate inputs produced of all types equals the aggregate human capital stock employed in the intermediate goods sector. That is,

$$\sum_{i=1}^{A_t} x_{it} = H_t^I. \tag{16}$$

Each of the ith intermediate good producer maximizes profits with respect to his/her choice of human capital. That is,

$$Max_{x_{it}} \;\; \pi_{it} = p_{it}x_{it} - w_I H_{it} = \alpha(H_t^Y)^{1-\alpha} x_{it}^{\alpha} - w_I x_{it},$$

where the expression in the right hand side derives from substituting the solution to p_{it} from eq. (13) and using $x_{it} = H_{it}$. w_I is wage rate prevailing in intermediate goods sector. The first-order condition yields:

$$w_I = \alpha^2 (H_t^Y)^{1-\alpha} x_{it}^{\alpha-1}. \tag{17}$$

Using eq. (13), we get the solution to the equilibrium price as $p_{it} = p_t = \frac{w_I}{\alpha}$. This is the monopoly price charged as a markup over marginal cost. Note that being independent of i, this price is constant across all intermediate goods. From eq. (15), this implies that the quantity produced of each ith intermediate input is the same, that is, $x_{it} = x_t = \left[\frac{\alpha^2}{w_I}\right]^{\frac{1}{1-\alpha}} H_t^Y$. This entails that, in equilibrium, the profit of the ith monopolist is given by:

$$\begin{aligned} \pi_t = p_t x_t - w_I x_t \equiv \left[\frac{w_I}{\alpha} - w_I\right] x_t \equiv \left[\frac{1-\alpha}{\alpha}\right] w_I x_t; \\ = \alpha(1-\alpha) H_Y^{1-\alpha} x_t^{\alpha}, \end{aligned} \tag{18}$$

where the last expression is derived by using eq. (17) and $x_{it} = x_t$ in equilibrium. Since, in equilibrium, intermediate inputs are sold at the same price and demanded in equal quantities, the aggregate human capital stock employed in the intermediate goods sector is given by $H_t^I = A_t x_t$. Inserting this information into the production function of the final good, eq. (12) simplifies to:

$$Y_t = (H_t^Y)^{1-\alpha} A_t^{1-\alpha} (H_t^I)^{\alpha}. \tag{19}$$

Accordingly, equilibrium profits of ith monopolist in eq. (18) simplify to yield:

$$\pi_t = \alpha(1-\alpha)\frac{Y_t}{A_t}. \tag{20}$$

Also, since in equilibrium, $x_{it} = x_t$ and $H_t^I = A_t x_t$, the wage rate in eq. (17) can be expressed as:

$$w_I = \alpha^2 (H_t^Y)^{1-\alpha} \left[\frac{A_t}{H_t^I}\right]^{1-\alpha}. \tag{21}$$

Further, using eq. (19), wage rate in intermediate goods sector simplifies to:

$$w_I = \alpha^2 \left[\frac{Y_t}{H_t^I}\right]. \tag{22}$$

Next, the R&D sector is discussed.

2.5 R&D Sector

Under the assumption of free entry and exit in the R&D sector, firms employ human capital or researchers, H_t^A, to develop new blueprints which are sold at price, p_t^A. This price is common to all the blueprints due to the competitive feature of the R&D sector. We consider two types of regimes that can drive R&D activities. The R&D sector produces blueprint of an intermediate variety either by imitating from the world technology frontier or by innovating upon the local technology level. Following Papageorgiou & Perez-Sebastian (2006) and Guilló et al. (2011), the production function of technology for a firm is postulated as:

$$A_{t+1} - A_t = \delta_t H_t^A, \tag{23}$$

where $A_{t+1} - A_t$ are new blueprints and H_t^A is the human capital working in R&D sector. Productivity of the R&D activity, δ_t, is constant at the firm level but at the aggregate level, it is defined as:

$$\text{Innovation regime} : \delta_t = \bar{\delta}(H_t^A)^{\lambda-1} A_t^{\phi}; \tag{24}$$

$$\text{Imitation regime} : \delta_t = \bar{\delta}(H_t^A)^{\lambda-1} A_t^{\phi} \left[\frac{\bar{A}_t}{A_t}\right].$$

Thus, R&D productivity, δ_t, evolves differently between the innovation and imitation regimes of technological development. R&D productivity depends positively on the number of already existing ideas, A_t, and human capital employed in R&D sector, H_t^A. The parameter $\bar{\delta}$ denotes general productivity in R&D. $0 < \phi < 1$ measures intertemporal knowledge spillovers (standing-on-shoulders effect) and $0 < \lambda < 1$ measures returns to R&D effort (stepping-on-toes effect). $\bar{A}_t$ is world technology frontier that grows exogenously at rate, $g_{\bar{A}}$. The standing-on-shoulders effect may arise as existing knowledge contributes to the capacity to innovate. The returns to human capital differ between the firm level and the economy-wide level. There exists constant returns to R&D effort at the firm level as revealed by eq. (23). However, the R&D technology shows diminishing returns to R&D effort as researchers generate negative externality at the aggregate level (stepping-on-toes effect). The stepping-on-toes effect may arise due to competition among multiple R&D firms to become the first to succeed in creating and patenting a new blueprint and/or process. [7]

If all other factors are held constant, an increase in R&D activity will induce increased duplication of research effort leading to stepping-on-toes effect. Additionally, R&D productivity depends on a catch-up term, $\frac{\bar{A}_t}{A_t}$ under the imitation regime. Akin to Nelson & Phelps (1966), $\frac{\bar{A}_t}{A_t}$ is the catch-up term, which signifies the fact that greater the technological gap between leader and follower economy, higher the potential of the follower economy to catch up through imitation of existing technologies. Since all R&D firms end up in a symmetric equilibrium, the production function of technology under innovation regime at the aggregate level reduces to:

$$A_{t+1} - A_t = \bar{\delta}(H_t^A)^{\lambda} A_t^{\phi}. \tag{25}$$

Under the imitation regime, the aggregate production function reduces to:

$$A_{t+1} - A_t = \bar{\delta}(H_t^A)^{\lambda} A_t^{\phi} \left[\frac{\bar{A}_t}{A_t}\right]. \tag{26}$$

The catch-up effect is specific to the imitation regime only. Each firm in the R&D sector maximizes profits, given by:

$$\pi_{t,A} = p_t^A (A_{t+1} - A_t) - w_A H_t^A,$$

where p_t^A is price of a blueprint, $A_{t+1} - A_t$ are number of new blueprints discovered and w_A is the wage rate. Under both imitation and innovation regimes, using eq. (23), the profit function of R&D firm can be expressed as:

$$\pi_{t,A} = p_t^A \delta_t (H_t^A) - w_A H_t^A. \tag{27}$$

In case of both the technology regimes, maximization of profits would lead to the following optimality condition:

$$w_A = p_t^A \delta_t. \tag{28}$$

Substituting for δ_t from eq. (24), the wage rate under innovation regime is now given by:

$$w_A^{in} = p_t^A \bar{\delta}(H_t^A)^{\lambda-1} A_t^{\phi} = \left[\frac{p_t^A \bar{\delta}(H_t^A)^{\lambda} A_t^{\phi}}{H_t^A}\right], \tag{29}$$

Similarly, wage rate under imitation regime is expressed as:

$$w_A^{im} = \frac{p_t^A \bar{\delta}(H_t^A)^{\lambda} A_t^{\phi} \frac{\bar{A}_t}{A_t}}{H_t^A}, \tag{30}$$

[7] The term, $(H_t^A)^{\lambda-1}$ in eq (24) captures the stepping-on-effect. There exists diminishing returns to R&D effort as $0 < \lambda < 1$. The standing-on-shoulders effect is captured by A_t^{ϕ} in eq. (24).

where superscripts 'in' and 'im' refer to variables under the innovation and imitation regimes. Using eqs. (25) and (26), the wage rate under both the technology regimes simplifies to:

$$w_A = \left[\frac{p_t^A(A_{t+1} - A_t)}{H_t^A}\right]. \tag{31}$$

where wages of researchers are increasing in the price of blueprint (price of patent) and number of blueprints discovered.

We, next, consider the research arbitrage condition. Shareholders of intermediate firms face two options. First, they can make an investment of p_t^A in a risk-free asset and earn the market rate of interest, r_t which is exogenously given. Alternatively, they can purchase shares of intermediate firms in period t and receive π_{t+1} as dividend in period t+1 and can sell these shares to next generation to earn capital gain/loss resulting from the change in price of patents over time. In equilibrium, the rate of return from both these investments should be the same. That is,

$$r_{t+1}p_t^A = \pi_{t+1} + (p_{t+1}^A - p_t^A).$$

The left hand side of this equation is the interest earned from investing in a risk-free asset. The right hand side is the sum of the dividend earned and the capital gain/loss.

3 Market clearing condition and Dynamics of the System

Human capital is used for production of final good, intermediate inputs and R&D activities. Now, in equilibrium, the demand for human capital in R&D sector, intermediate inputs sector and final good sector should add up to:

$$H_t^A + H_t^Y + H_t^I = H_t. \tag{32}$$

This gives the labor market clearing condition.

Next, we consider the equilibrium condition for final goods market. Final good is used for consumption and for incurring education expenditure on children. The final goods market clearing condition is given by:

$$Y_t = c_{1,t}N_t + c_{2,t}N_{t-1} + E_t \tag{33}$$

where $E_t = e_t w_t h_t n_t N_t$ is the total education expenditure incurred on next generation by the present generation.

Furthermore, the aggregate savings of young adults in period t must be used for net investment in R&D activities i.e.

$$p_t^A(A_{t+1} - A_t) = s_t N_t - p_t^A A_t$$

The term on left-hand side is the net investment in R&D activities and the first term on right-hand side is aggregate savings of young adults and second term is the dissavings of the old and the two terms together on the right-hand side represent the net savings. Eliminating $p_t^A A_t$ from both the sides yields the following asset market clearing condition[8]:

$$p_t^A A_{t+1} = s_t N_t \tag{34}$$

In equilibrium, wages in final good, intermediates and R&D sectors should equalize, that is, $w_Y = w_A = w_I$. Let that equalized wage rate be w_t. Substituting for w_Y and w_I from eqs. (14) and (22), we get that:

$$\frac{H_t^I}{H_t^Y} = \frac{\alpha^2}{1-\alpha} \tag{35}$$

Now, $H_t^Y + H_t^I = H_t - H_t^A$. Substituting for H_t^Y from eq.(35), we derive the human capital stock engaged in intermediates sector as:

$$H_t^I = \frac{\alpha^2}{1-\alpha+\alpha^2}[H_t - H_t^A] \tag{36}$$

Next, we substitute for s_t from eq.(6) and p_t^A from eq. (28) in the asset market clearing condition and use the fact that $w_Y = w_A = w_I = w_t$ at equilibirum and that the size of the workforce or supply of labor is given by $L_t = (1-\tau n_t)N_t$ to get[9]:

$$A_{t+1} = \zeta \delta_t H_t \tag{37}$$

where $\zeta = \frac{\beta_1}{1+\beta_1+\beta_2(1-\tau n_t)}$. Now, we get the following expression for A_{t+1} under the two technology regimes after substituting for δ_t from eq. (24):

$$\text{Innovation regime} : A_{t+1} = \zeta\bar{\delta}(H_t^A)^\lambda A_t^\phi \frac{H_t}{H_t^A}; \tag{38}$$

$$\text{Imitation regime} : A_{t+1} = \zeta\bar{\delta}(H_t^A)^\lambda A_t^\phi \left[\frac{\bar{A}_t}{A_t}\right] \frac{H_t}{H_t^A}.$$

which after substituting from eqs. (25) and (26), simplifies to a single expression for both the regimes:

$$\frac{H_t^A}{H_t} A_{t+1} = \zeta(A_{t+1} - A_t)$$

[8]Detailed derivation of asset market clearing condition is provided in Appendix B

[9]We can treat n_t as a constant as we know from eq. (8) that fertility rate is constant over time

We get the employment share in R&D sector for both technology regimes after dividing both sides by A_t as

$$\frac{H_t^A}{H_t} = \zeta \frac{g_{A,t}}{1 + g_{A,t}} \tag{39}$$

Accordingly, the equilibrium wage rate in the three sectors is given by:

$$w_A = w_Y = w_I = \frac{(1-\alpha)Y_t}{H_t^Y}. \tag{40}$$

We, next, examine the dynamic properties of our stylized economy. First, we discuss the dynamics of physical factors of production. The aggregate population, N_t, grows at the fertility rate, n_t as follows:

$$N_{t+1} = n_t N_t, \tag{41}$$

where n_t is constant and endogenously given by eq. (8).

Taking child rearing time into account, the size of the workforce or supply of labor is given by $L_t = (1 - \tau n_t)N_t$. Since child rearing costs are constant over time, and from eq. (8) we know that fertility rate is also constant over time, the workforce grows at the fertility rate as:

$$L_{t+1} = n_t L_t. \tag{42}$$

Next, we discuss the dynamics of aggregate human capital, $H_t \equiv h_t L_t$. The dynamics of per capita human capital are given by eq. (9). Using eqs. (9) and (42), the equation for aggregate human capital accumulation can be written as:

$$\frac{H_{t+1}}{H_t} = \begin{cases} \mu^{\epsilon} n_t, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon}; \\ \left[\frac{\epsilon(\tau\theta - \mu)}{(1-\epsilon)}\right]^{\epsilon} n_t, & \text{otherwise.} \end{cases} \tag{43}$$

The dynamics of total factor productivity are given by eq.(38) as:
Innovation regime:

$$A_{t+1} = \zeta\bar{\delta}(H_t^A)^{\lambda} A_t^{\phi} \frac{H_t}{H_t^A}. \tag{44}$$

Imitation regime:

$$A_{t+1} = \zeta\bar{\delta}(H_t^A)^{\lambda} A_t^{\phi} \left[\frac{\bar{A}_t}{A_t}\right] \frac{H_t}{H_t^A}. \tag{45}$$

Using eqs.(19),(35) and (36), the dynamics of aggregate output can be written as:

$$Y_t = \frac{(1-\alpha)^{1-\alpha}\alpha^2}{\alpha^{2(1-\alpha)}(1-\alpha+\alpha^2)} A_t^{1-\alpha} \left[1 - \frac{H_t^A}{H_t}\right] H_t \tag{46}$$

The system of equations,(eqs. (41)-(46)) fully describes the equilibrium dynamics of our model economy for all the plausible cases. The next section characterizes the balanced growth paths of an economy for two cases - a) when the economy's quality of education system is sufficiently high, that is, $\theta > \frac{\mu}{\tau\epsilon}$, and b) when quality of schooling is less than the threshold, or, $\theta \leq \frac{\mu}{\tau\epsilon}$.

4 Balanced Growth Path and Steady-State Properties of the Stylized Economy

4.1 Characterizing the Balanced Growth Path

We denote the growth rate of x along the balanced growth path by g_x, that is, by omitting the time index for brevity. [10]

We, first, consider the growth rate of human capital accumulation. The proportion of work-force employed in final goods, intermediates and R&D sectors, (given by eqs. (35),(36), (39) respectively) are constant as $g_{A,t}$ is constant along the balanced growth path. Therefore, along the balanced growth path, we have:

$$\frac{H^Y_{t+1}}{H^Y_t} = \frac{H^A_{t+1}}{H^A_t} = \frac{H^I_{t+1}}{H^I_t} = \frac{H_{t+1}}{H_t} = (1 + g_H). \tag{47}$$

Thus, the human capital stocks in the final good, intermediates and the R&D sectors grow at the rate of total human capital accumulation along the balanced growth path.

Next, we consider the growth rate of total factor productivity. Under innovation regime, we observe from eq. (44) that:

$$(1 + g_{A,t}) = \frac{\zeta\bar{\delta}H_t(H^A_t)^{\lambda-1}}{A_t^{1-\phi}};$$

Since along balanced growth path, the left hand side is constant, therefore, right hand side must also be constant and this holds true when

$$(1 + g_A) = [(1 + g_h)n]^{\frac{\lambda}{1-\phi}}. \tag{48}$$

The right hand side follows from the definition of aggregate human capital $H_t = h_t L_t$ and from eq. (47). Further, we observe from eq. (45) that the rate of technical progress under the imitation regime can be written as:

$$(1 + g_{A,t}) = \frac{\zeta\bar{\delta}H_t\bar{A}_t(H^A_t)^{\lambda-1}}{A_t^{2-\phi}}.$$

[10] A balanced growth path is a long run equilibrium of the economy, also defined as the steady state, along which growth rate of variables is either zero or constant over time. For any variable x, the growth rate is denoted by $g_{x,t} = (x_{t+1} - x_t)/x_t$, and its rate of change by $\tilde{g}_{x,t} = (g_{x,t+1} - g_{x,t})/g_{x,t}$. The balanced growth, thus, requires $\tilde{g}_{x,t} = 0$.

Similarly, using the definition of the balanced growth path, we derive the long-run rate of technological progress under imitation regime as:

$$(1 + g_A) = (1 + g_H)^{\frac{\lambda}{2-\phi}}(1 + g_{\bar{A}})^{\frac{1}{2-\phi}} = [(1 + g_h)n]^{\frac{\lambda}{2-\phi}}(1 + g_{\bar{A}})^{\frac{1}{2-\phi}}. \tag{49}$$

Thus, intuitively, under both imitation and innovation regimes, technological progress is driven by growth in aggregate human capital. Human capital accumulation improves productivity of researchers, which fosters technological progress. Besides aggregate human capital, the growth of world technology frontier is also a driver of growth, but only in case of imitation regime. The follower economy takes advantage of existing technologies through technology adoption. Therefore, as the world technology frontier grows, it enhances the potential of the follower country to catch up through imitation.

Next, we ascertain the growth rates of aggregate output and per capita consumption along the balanced growth path. From eq. (46) we observe that:

$$(1 + g_Y) = (1 + g_A)^{1-\alpha}(1 + g_H). \tag{50}$$

Along the balanced growth path, growth rate of aggregate output depends on growth rate of human capital and total factor productivity.

From eqs. (48), (49) and (50), we derive the balanced growth path of the stylized economy under the two technology regimes as:
Innovation regime:

$$(1 + g_Y) = [(1 + g_h)n]^{\frac{1-\phi+\lambda(1-\alpha)}{1-\phi}}; \tag{51}$$

Imitation regime:

$$(1 + g_Y) = [(1 + g_h)n]^{\frac{2-\phi+\lambda(1-\alpha)}{2-\phi}}(1 + g_{\bar{A}})^{\frac{1}{2-\phi}}, \tag{52}$$

where

$$[(1 + g_h)n] = (1 + g_H) = \begin{cases} \dfrac{\beta_2\theta\epsilon}{(1 + \beta_1 + \beta_2)\mu^{1-\epsilon}}, & \text{if } \theta < \dfrac{\mu}{\tau\epsilon}; \\ \dfrac{\beta_2\mu^{\epsilon}}{(1 + \beta_1 + \beta_2)\tau}, & \text{if } \theta = \dfrac{\mu}{\tau\epsilon}; \\ \dfrac{\beta_2\theta\epsilon^{\epsilon}(1-\epsilon)^{1-\epsilon}}{(1 + \beta_1 + \beta_2)(\tau\theta - \mu)^{1-\epsilon}}, & \text{otherwise.} \end{cases}$$

This follows after substituting the value of n from eq. (8) into eq. (43). Thus, along the steady state, the growth rates of aggregate output are determined by the rate of human capital accumulation under both the regimes of technical improvement.

Furthermore, we observe from the consumer's optimization exercise that the Euler equation has the following standard form:

$$\frac{c_{t+1}}{c_t} = \beta_1(1 + r_{t+1}). \tag{53}$$

The right hand side follows from substituting values of c_t and s_t from eqs. (5) and (6) in eq. (3). Along the balanced growth path, per capita consumption grows at a constant rate under both the technology regimes as r_t is exogenously given.

We next compare the economic growth rates under the two technology regimes. We denote the economic growth rate under innovation regime by g_Y^{inn} and under imitation regime by g_Y^{imi}. The innovation economy will grow at a higher rate if

$$g_Y^{inn} > g_Y^{imi}.$$

Substituting for g_Y^{inn} and g_Y^{imi} from eqs. (51) and (52), we get:

$$(1 + g_H)^{\frac{1-\phi+\lambda(1-\alpha)}{1-\phi}} > (1 + g_H)^{\frac{2-\phi+\lambda(1-\alpha)}{2-\phi}}(1 + g_{\bar{A}})^{\frac{1}{2-\phi}}$$

which on simplification yields:[11]

$$(1 + g_H)^{\frac{\lambda(1-\alpha)}{1-\phi}} > (1 + g_{\bar{A}}).$$

Thus, we have,

Proposition 4.1 *(i) Under innovation regime, total factor productivity, aggregate output and per capita consumption grow at a constant rate along the balanced growth path characterized by eqs. (48), (51) and (53).*

(ii) Under imitation regime, total factor productivity, aggregate output and per capita consumption grow at a constant rate along the balanced growth path characterized by eqs. (49), (52) and (53).

(iii) Along the balanced growth path, the economy under innovation regime exhibits a higher economic growth rate as compared to imitation regime if

$$g_H > g_{\bar{A}}, \quad \lambda + \phi \leq 1$$

Intuitively, under both the technology regimes, the self-sustaining endogenous growth path is driven by human capital accumulation when quality of schooling exceeds the threshold, $\theta > \frac{\mu}{\tau\epsilon}$. In this case, at the micro level, parents decide to have fewer number of children and invest more

[11] We get the same condition if we compare per capita economic growth rates under the two technology regimes.

in their education. This follows from Lemma 1. At the macro level, this trade-off raises the rate of human capital accumulation, which encourages faster technological progress and, therefore, economic growth. Besides human capital, growth of world technology frontier is an additional driver of growth under the imitation regime via the catch-up effect.

Alternatively, when quality of schooling is less than the threshold, $\theta \leq \frac{\mu}{\tau\epsilon}$, parents do not invest in the education of children and instead, maximize fertility. In this case, the balanced growth path of the economy is driven only by population growth, which in turn, is determined by the fertility rate. This result is similar to the findings of neo-classical models of Solow-Swan and Cass-Koopmans-Ramsey. The only difference is that population growth and technical progress are endogenously determined in our model structure.

Thus, the drivers of economic growth differ depending upon the level of quality of schooling. When quality of schooling surpasses the threshold level, economic growth is driven by human capital accumulation whereas it is driven by population growth when quality of schooling is less than the threshold. In a similar context, while Eckstein & Zilcha (1994) do not specifically model the quality of schooling in their OLG framework, they show the effect of compulsory education on economic growth and distribution. Their analysis reveals that compulsory schooling financed by proportional taxes on income increases economic growth, and makes income distribution more equitable in the long-run. Similarly, Tamura (2001) explicitly model quality of schooling in their model of human capital accumulation but does not consider technical progress. He shows that quality of education fosters human capital formation and, therefore, economic growth.

Also, it is advantageous for an economy to innovate upon the local technology frontier instead of imitating from the world technology frontier if the rate of human capital accumulation is higher than the growth rate of world technology frontier in the presence of constant or diminishing returns to R&D sector (i.e. $\lambda + \phi \leq 1$). This implies that an economy has the potential to become the new world technology frontier if quality of schooling should be sufficiently high such that it leads to high enough investment in the education of children so that human capital accumulation emerges as a driver of economic growth.

We next turn to characterizing the evolution of wage rate along the steady state. It is known from eq. (40) that wage rate can be expressed as:

$$w_A = w_Y = w_I = \frac{(1-\alpha)Y_t}{H_t^Y}.$$

Further, from eq. (51), under the innovation regime:

$$(1+g_Y) = [(1+g_h)n]^{\frac{1-\phi+\lambda(1-\alpha)}{1-\phi}}, and$$

from eq. (52), under imitation regime, we have:

$$(1+g_Y) = [(1+g_h)n]^{\frac{2-\phi+\lambda(1-\alpha)}{2-\phi}}(1+g_{\bar{A}})^{\frac{1}{2-\phi}}.$$

Eqs. (40), (47), (51) and (52) together imply that, along balanced growth path, wage rate under innovation regime grows as follows:

$$(1+g_w) = [(1+g_h)n]^{\frac{\lambda(1-\alpha)}{1-\phi}}; \tag{54}$$

And the wage rate under imitation regime grows at the rate:

$$(1+g_w) = [(1+g_h)n]^{\frac{\lambda(1-\alpha)}{2-\phi}}(1+g_{\bar{A}})^{\frac{1}{2-\phi}}, \tag{55}$$

where

$$[(1+g_h)n] = (1+g_H) = \begin{cases} \dfrac{\beta_2\theta\epsilon}{(1+\beta_1+\beta_2)\mu^{1-\epsilon}}, & \text{if } \theta < \dfrac{\mu}{\tau\epsilon}; \\ \dfrac{\beta_2\mu^{\epsilon}}{(1+\beta_1+\beta_2)\tau}, & \text{if } \theta = \dfrac{\mu}{\tau\epsilon}; \\ \dfrac{\beta_2\theta\epsilon^{\epsilon}(1-\epsilon)^{1-\epsilon}}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^{1-\epsilon}}, & \text{otherwise} \end{cases}.$$

Intuitively, the wage rate depends on aggregate output and human capital as expressed by eq. (40). It is known from eq. (50) that growth rate of aggregate output or GDP depends on growth rate of human capital and total factor productivity along the balanced growth path. Therefore, the wage rate grows at the rate of technical progress along the balanced growth path under both the regimes of technological improvement.

Next, we compare the two cases of high and low quality of schooling and determine the condition under which the economy exhibits higher growth rate of per capita income under the case of higher quality of schooling, $\theta > \frac{\mu}{\tau\epsilon}$, as compared to the case of low quality of schooling, $\theta \leq \frac{\mu}{\tau\epsilon}$, under the two technology regimes.

4.2 Comparative Analysis of Per Capita Economic Growth Rates of Economies with Higher and Lower Quality of Schooling

We assume that when $\theta > \frac{\mu}{\tau\epsilon}$, quality of schooling is denoted by θ_h for that particular economy whereas quality of schooling is denoted by θ_l for an economy with quality of schooling less than the threshold, $\theta \leq \frac{\mu}{\tau\epsilon}$. Derivations provided in Appendix C show that an economy with higher quality of schooling, θ_h, will experience a higher per capita income growth rate as compared to an economy with a lower quality of schooling, θ_l, if the following conditions hold under the individual technology regimes.

Innovation regime:

$$\theta_h > \theta_l\left[\frac{(\tau\theta_h-\mu)\epsilon}{\mu(1-\epsilon)}\right]^{\frac{\lambda(1-\alpha)-\epsilon(1-\phi+\lambda(1-\alpha))}{\lambda(1-\alpha)}}; \tag{56}$$

Imitation regime:

$$\theta_h > \theta_l \left[\frac{(\tau\theta_h - \mu)\epsilon}{\mu(1-\epsilon)} \right]^{\frac{\lambda(1-\alpha)-\epsilon(2-\phi+\lambda(1-\alpha))}{\lambda(1-\alpha)}}. \tag{57}$$

Thus, we have,

Proposition 4.2 *An economy with higher quality of schooling,* $\theta_h > \frac{\mu}{\tau\epsilon}$*, will experience a higher per capita income growth rate as compared to an economy with lower quality of schooling,* $\theta_l \leq \frac{\mu}{\tau\epsilon}$ *if the quality of schooling,* θ_h*, is sufficiently high captured by the following parametric restriction:*

$$\text{Innovation regime: } \theta_h > \theta_l \left[\frac{(\tau\theta_h - \mu)\epsilon}{\mu(1-\epsilon)} \right]^{\frac{\lambda(1-\alpha)-\epsilon(1-\phi+\lambda(1-\alpha))}{\lambda(1-\alpha)}} > \theta_l; \tag{58}$$

$$\text{Imitation regime: } \theta_h > \theta_l \left[\frac{(\tau\theta_h - \mu)\epsilon}{\mu(1-\epsilon)} \right]^{\frac{\lambda(1-\alpha)-\epsilon(2-\phi+\lambda(1-\alpha))}{\lambda(1-\alpha)}} > \theta_l.$$

Intuitively, a mere surpassing of the threshold level of quality schooling is not sufficient enough for an economy to experience a higher growth rate of per capita output as compared to an economy with quality of schooling lower than the threshold level. Under the two technology regimes, quality of schooling should be sufficiently high as indicated by the eq. (58) such that it leads to high enough investment in the education of children, entailing that the growth-stimulating effect overpowers the growth-impeding effect of quality of schooling.

Otherwise, the possibility that an economy with lower quality of schooling experiences a higher per capita economic growth rate than an economy with higher quality of schooling is not ruled out, especially for large enough values of child rearing costs, τ or for small enough value of inter-generational human capital spillovers, μ and returns to education, ϵ respectively. This follows directly from eq. (58). It can be observed that the expression, $\frac{(\tau\theta_h - \mu)\epsilon}{\mu(1-\epsilon)}$ is increasing in τ. In this particular case, when the value of τ is sufficiently high, population growth rate may turn out to be a more effective driver of economic growth as the threshold value of quality of schooling for higher economic growth is so high that an economy may find not investing at all in education of the future generation as a relatively more beneficial outcome. Similarly, it can be shown that:

$$\frac{\partial}{\partial\mu}\frac{(\tau\theta_h - \mu)}{\mu} = \frac{-\tau\theta_h}{\mu^2} < 0;$$
$$\frac{\partial}{\partial\epsilon}\frac{\epsilon}{1-\epsilon} = \frac{-1}{(1-\epsilon)^2} < 0.$$

These imply that the threshold value of quality of schooling for higher per capita economic growth is decreasing in the value of μ and ϵ respectively. Thus, this threshold value of quality of schooling can be high enough for sufficiently small μ and ϵ such that population growth rate may turn out to be a more effective driver of economic growth and an economy or an individual may not invest in human capital of its future generation.

This result is similar to the empirical findings of Castelló-Climent & Hidalgo-Cabrillana (2012) and Hanushek & Woessmann (2012). Castelló-Climent & Hidalgo-Cabrillana (2012) find that the positive effect of schooling quality on growth is found only when it is relatively high, which leads to the suggestion that schooling quality is not growth enhancing unless students achieve a minimum level of knowledge. Hanushek & Woessmann (2012), further show that effect of quality of schooling (proxied by share of high achieving students) is significantly larger in countries that have more scope to catch up to the most technologically advanced countries.[12] Altinok & Aydemir (2017) also find that the share of top performers in student achievement tests has a strong and positive effect on economic growth in high income countries. Intuitively, many developing countries frequently face the trade-off between spending resources on expanding basic access to education and spending it on those students identified as the best (high achievers) given resource constraints. This also amounts to the quantity versus quality trade-off. From this perspective, countries need human capital with high cognitive skills for an imitation strategy, and the process of economic convergence is accelerated in countries with larger shares of high performing students.

Furthermore, eqs. (51) and (52) suggest that technological progress and aggregate output are positively correlated with population growth. This implies that a decline in population growth entails a lowering of rate of technical progress as postulated by conventional R&D based growth models (Romer 1990, Jones 1995). This type of macro-level examination tends to miss the point that aggregate human capital accumulation and fertility rate are inversely related via quality-quantity trade-off at the family/household level, as shown in Lemmas 1 and 2. The investment in education increases and fertility rate falls simultaneously as the quality of schooling increases above the threshold. This quality-quantity trade-off implies that the growth rate of population falls whereas the growth rate of human capital rises as the quality of schooling increases above the threshold. This leads to the question: how do improvement in quality of schooling and returns to education affect total factor productivity growth and, therefore, per capita economic growth by influencing fertility and education decisions?

The answer to these questions could be found by carrying out comparative dynamics with respect to parameters related to quality of schooling and returns to education. This is attempted in the next subsection.

[12] Hanushek & Woessmann (2012) look at the distribution of scores by defining two variables that measure the proportion of students that meet a threshold level of achievement. The first was a score of 400 or above on the transformed international scale, that is, one standard deviation below the mean test scores for OECD countries (meant to capture basic literacy) and the other 600 or above (to capture high achievement).

4.3 Comparative Dynamics

4.3.1 Comparative Dynamics w.r.t Quality of Schooling, θ

Since total factor productivity growth and per capita economic growth depend on the rate of human capital accumulation under both the regimes of technological improvement, we first carry out comparative dynamics of growth rate of aggregate human capital with respect to schooling quality. From eq. (52), we know that $(1 + g_H) = (1 + g_h).n$. Differentiating both the sides w.r.t θ yields:

$$\frac{\partial g_H}{\partial \theta} = (1 + g_h)\frac{\partial n}{\partial \theta} + n\frac{\partial g_h}{\partial \theta}. \tag{59}$$

When schooling quality, θ, exceeds the threshold, that is, $\theta > \frac{\mu}{\tau\epsilon}$, $\frac{\partial g_H}{\partial \theta}$ is given by:[13]

$$\begin{aligned}\frac{\partial g_H}{\partial \theta} &= (1 + g_h)\left[\frac{\epsilon\tau\beta_2\theta(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2} - \frac{\mu\beta_2(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2}\right] \\ &= \left[\frac{(1+g_h)\beta_2(\epsilon\tau\theta-\mu)(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2}\right] > 0,\end{aligned} \tag{60}$$

in view of $\epsilon < 1$.

We next analyze the derivative of the growth rate of aggregate human capital with respect to schooling quality when it is less than the threshold, that is, $\theta < \frac{\mu}{\tau\epsilon}$[14]. We know from Lemma 1 that $\frac{\partial n_t}{\partial \theta} = \frac{\beta_2\epsilon}{(1+\beta_1+\beta_2)\mu}$, and from eq. (9), we have $(1 + g_h) = \mu^{\epsilon}$. Differentiating g_h w.r.t θ yields:

$$\frac{\partial g_h}{\partial \theta} = 0. \tag{61}$$

Substituting this into eq. (59), we get that:

$$\frac{\partial g_H}{\partial \theta} = \left[\frac{\beta_2\epsilon}{(1+\beta_1+\beta_2)\mu^{1-\epsilon}}\right] > 0. \tag{62}$$

We next consider the comparative dynamics of growth rate of per capita output, $g_y = Y_t/L_t$ with respect to θ. At steady state, growth rate of per capita output under innovation regime is given by:

$$g_y = (1 + g_h)^{\frac{1-\phi+\lambda(1-\alpha)}{1-\phi}} n^{\frac{\lambda(1-\alpha)}{1-\phi}} - 1. \tag{63}$$

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[13]Detailed derivations of eq. (60) are provided in Appendix D.

[14]We know from Lemma 1 that quality of schooling has no effect on fertility when $\theta = \frac{\mu}{\tau\epsilon}$, i.e. $\frac{\partial e_t}{\partial \theta} = \frac{\partial n_t}{\partial \theta} = 0$. Therefore, we carry out comparative dynamics of growth rate of aggregate human capital and per capita output growth with respect to schooling quality only for the case,$\theta < \frac{\mu}{\tau\epsilon}$.

And under imitation regime, it is given by:

$$g_y = (1 + g_{\bar{A}})^{\frac{1}{2-\phi}} (1 + g_h)^{\frac{2-\phi+\lambda(1-\alpha)}{2-\phi}} n^{\frac{\lambda(1-\alpha)}{2-\phi}} - 1. \tag{64}$$

We first consider the case when, $\theta > \dfrac{\mu}{\tau\epsilon}$. Differentiating g_y with respect to θ under innovation regime yields:[15]

$$\frac{\partial g_y}{\partial \theta} = \frac{1 + g_y}{1 - \phi}\left[\frac{(1 - \phi + \lambda(1 - \alpha))\tau\epsilon}{\tau\theta - \mu} - \frac{\mu\lambda(1 - \alpha)}{\theta(\tau\theta - \mu)}\right] > 0. \tag{65}$$

Similarly, under imitation regime, differentiating per capita income growth rate with respect to θ yields:

$$\frac{\partial g_y}{\partial \theta} = \frac{1 + g_y}{2 - \phi}\left[\frac{(2 - \phi + \lambda(1 - \alpha))\tau\epsilon}{\tau\theta - \mu} - \frac{\mu\lambda(1 - \alpha)}{\theta(\tau\theta - \mu)}\right] > 0. \tag{66}$$

When $\theta < \dfrac{\mu}{\tau\epsilon}$, differentiating per capita income growth rate with respect to θ yields the following results:[16]

In innovation regime,

$$\frac{\partial g_y}{\partial \theta} = \frac{(1 + g_y)\lambda(1 - \alpha)}{\theta(1 - \phi)} > 0; \tag{67}$$

In imitation regime:

$$\frac{\partial g_y}{\partial \theta} = \frac{(1 + g_y)\lambda(1 - \alpha)}{\theta(2 - \phi)} > 0, \tag{68}$$

as $\phi < 1$. Thus, from eqs. (60), (62), (65), (66), (67) and (68), it can be deduced that,

Proposition 4.3 *The long-run rate of technical progress, g_A, and per capita economic growth, g_y, increase in response to an improvement in the quality of schooling on account of different channels, depending upon the quality of schooling, θ:*

(i) When $\theta > \dfrac{\mu}{\tau\epsilon}$, g_A and g_y are increasing in θ (that is, $\dfrac{\partial g_A}{\partial \theta} > 0$ and $\dfrac{\partial g_y}{\partial \theta} > 0$) due to higher rate of human capital accumulation under both the regimes of technological improvement.

(ii) When $\theta < \dfrac{\mu}{\tau\epsilon}$, g_A and g_y are increasing in θ (that is, $\dfrac{\partial g_A}{\partial \theta} > 0$ and $\dfrac{\partial g_y}{\partial \theta} > 0$) due to higher population growth rate under both the regimes of technological improvement and g_A and g_y do not depend on θ when $\theta = \dfrac{\mu}{\tau\epsilon}$

[15] Detailed derivations of eqs. (65) and (66) are provided in Appendix E.

[16] Detailed derivations of eqs. (67) and (68) are also provided in Appendix E.

The intuitive explanation for the impact of a change in quality of schooling on the long-run rate of technical progress and per capita economic growth is as follows. When quality of schooling surpasses the threshold, it has two opposing effects on human capital accumulation. We know from Lemma 1 that an improvement in quality of schooling increases investment in the education of a child. This stimulates the accumulation of human capital which fosters technical progress leading to higher economic growth in the economy. This effect can be regarded as the growth-stimulating effect. The increase in education is also accompanied by a decline in fertility rate as the quality of education improves. This constitutes the growth-impeding effect that reduces total factor productivity growth and economic growth by contracting the pool of available researchers.

$$\frac{\partial g_H}{\partial \theta} = (1+g_h)\left[\underbrace{\frac{\epsilon\tau\beta_2\theta(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2}}_{\text{Growth-stimulating effect}} - \underbrace{\frac{\mu\beta_2(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2}}_{\text{Growth-impeding effect}}\right]$$

Total factor productivity growth and economic growth will accelerate or decelerate depending upon the relative magnitudes of these two effects. As shown in eq. (60), the growth-stimulating effect overpowers the growth-impeding effect of a change in quality of schooling when quality of schooling exceeds the threshold, that is, $\theta > \frac{\mu}{\tau\epsilon}$. Thus, in the aggregate, the growth rate of technology increases in response to an increase in schooling quality that sustains economic growth in the long-run.

Also, the growth rate of per capita income along the balanced growth path can be expressed as $(1+g_y) = \frac{(1+g_Y)}{n} = \frac{(1+g_H)(1+g_A)^{(1-\alpha)}}{n}$. The economic growth rate, g_Y, is increasing in θ as growth-stimulating effect dominates the growth-impeding effect of quality of schooling when it exceeds the threshold. Also, it is known from Lemma 1 that parents bear a lower number of children in response to an improvement in quality of schooling. Thus, the fertility rate or the population growth rate is decreasing in θ. Consequently, the growth rate of per capita income rises as quality of schooling improves under both the technology regimes.

When quality of schooling is strictly less than the threshold, it is known from Lemma 1 that parents do not educate their children and instead focus on having more children. In this particular case, there exist no growth-stimulating and growth-impeding effects of quality of schooling on aggregate human capital. Instead, the rate of technical progress increase in response to an increase in the quality of schooling solely due to higher population growth, as parents focus on maximizing fertility when quality of schooling is less than the threshold.

However, it can be observed from eqs. (63) and (64) that quality of schooling raises population growth rate by a lesser proportion as compared to the proportionate rise in growth rate of aggregate output under both the regimes when quality of schooling is less than the threshold. As a result, growth rate of per capita income is increasing in quality of schooling under both the technology regimes.

Thus, contingent upon the quality of schooling, there are two different channels at work which foster technical progress and economic growth. When quality of schooling exceeds the threshold, rate of human capital accumulation is the driver of economic growth whereas population growth rate drives economic growth when quality of schooling is strictly lower than the threshold. This result is similar to that of Hashimoto & Tabata (2016) about old-age survival probability and economic growth. They find that in economies where old-age survival probability is sufficiently low, an increase in old-age survival probability motivates individuals to invest more in their own education, thus, accelerating the accumulation of per capita human capital and, thereby, enhancing the long-run growth rate of the economy. However, in economies where old-age survival probability is sufficiently high, an increase in old age survival probability will lead to a decline in population growth rates, thereby, lowering the long-run growth rate of the economy.

Next, comparative dynamics with respect to returns to education, ϵ, are discussed.

4.3.2 Comparative Dynamics w.r.t Returns to Education, ϵ

We first analyze the derivative of the growth rate of aggregate human capital with respect to returns to education. We know that:

$$(1 + g_H) = (1 + g_h).n$$

Differentiating both the sides w.r.t ϵ, yields:

$$\frac{\partial g_H}{\partial \epsilon} = (1 + g_h)\frac{\partial n}{\partial \epsilon} + n\frac{\partial g_h}{\partial \epsilon} \tag{69}$$

We get that,[17]

$$\frac{\partial g_H}{\partial \epsilon} = (1 + g_h)n\left[\left[\frac{1}{(1-\epsilon)} + log\frac{\epsilon(\tau\theta - \mu)}{1-\epsilon}\right] - \frac{1}{1-\epsilon}\right] = (1 + g_H)log\left[\frac{\epsilon(\tau\theta - \mu)}{1-\epsilon}\right] > 0 \tag{70}$$

as $\dfrac{(\tau\theta - \mu)\epsilon}{1-\epsilon} > 1$ from eq. (C.6).

Similarly, when $\theta \leq \dfrac{\mu}{\tau\epsilon}$, the derivative of the growth rate of aggregate human capital with respect to returns to education yields:

$$\frac{\partial g_H}{\partial \epsilon} = \begin{cases} (1 + g_H)\left[\frac{1}{\epsilon} + log\mu\right] > 0, & \text{if } \theta < \frac{\mu}{\tau\epsilon}, \\ (1 + g_H)\left[log\mu\right] > 0, & \theta = \frac{\mu}{\tau\epsilon}. \end{cases} \tag{71}$$

Lastly, we examine the impact of a change in returns to education, ϵ, on the growth rate of per capita income along the balanced growth path under the two technology regimes. When $\theta > \dfrac{\mu}{\tau\epsilon}$, differentiating per capita income growth rate with respect to ϵ under the innovation and imitation regimes, yields:[18]

[17] Detailed derivations of eqs. (70) and (71) are provided in Appendix F.

[18] Detailed derivations of eqs. (72) and (73) are provided in Appendix G.

Innovation Regime:

$$\frac{\partial g_y}{\partial \epsilon} = \frac{(1-\phi+\lambda(1-\alpha))(1+g_y)}{1-\phi}\left[\frac{1}{1-\epsilon} + log\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon}\right] - \frac{(1+g_y)\lambda(1-\alpha)}{(1-\epsilon)(1-\phi)} > 0 \tag{72}$$

Similarly, under imitation regime, we have,

$$\frac{\partial g_y}{\partial \epsilon} = \frac{(2-\phi+\lambda(1-\alpha))(1+g_y)}{2-\phi}\left[\frac{1}{1-\epsilon} + log\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon}\right] - \frac{(1+g_y)\lambda(1-\alpha)}{(1-\epsilon)(2-\phi)} > 0. \tag{73}$$

We next, consider the case when $\theta \leq \frac{\mu}{\tau\epsilon}$. Differentiating per capita income growth rate with respect to ϵ yields the following.[19]
Innovation regime:

$$\frac{\partial g_y}{\partial \epsilon} = \begin{cases} (1+g_y)\left[1+\frac{\lambda(1-\alpha)}{1-\phi}\right] log\mu + \frac{\lambda(1-\alpha)(1+g_y)}{(1-\phi)\epsilon} > 0; & \text{if } \theta < \frac{\mu}{\tau\epsilon}, \\ (1+g_y)\left[1+\frac{\lambda(1-\alpha)}{1-\phi}\right] log\mu > 0, & \theta = \frac{\mu}{\tau\epsilon} \text{ and} \end{cases} \tag{74}$$

Imitation regime:

$$\frac{\partial g_y}{\partial \epsilon} = \begin{cases} (1+g_y)\left[1+\frac{\lambda(1-\alpha)}{2-\phi}\right] log\mu + \frac{\lambda(1-\alpha)(1+g_y)}{(2-\phi)\epsilon} > 0; & \text{if } \theta < \frac{\mu}{\tau\epsilon}, \\ (1+g_y)\left[1+\frac{\lambda(1-\alpha)}{2-\phi}\right] log\mu > 0, & \theta = \frac{\mu}{\tau\epsilon}. \end{cases} \tag{75}$$

since $\phi < 1$ and $\mu \geq 1$. An examination of these derivatives yields the following results.

Proposition 4.4 *The long-run rate of technical progress, g_A, and per capita aggregate output, g_y, increase in response to an increase in returns to education, ϵ, on account of different channels, depending upon the quality of schooling, θ:*

(i) When $\theta > \frac{\mu}{\tau\epsilon}$, g_A and g_y are increasing in ϵ (that is, $\frac{\partial g_A}{\partial \epsilon} > 0$ and $\frac{\partial g_y}{\partial \epsilon} > 0$) due to higher rate of human capital accumulation under both the regimes of technological improvement.

(ii) When $\theta < \frac{\mu}{\tau\epsilon}$, g_A and g_y are increasing in ϵ (that is, $\frac{\partial g_A}{\partial \epsilon} > 0$ and $\frac{\partial g_y}{\partial \epsilon} > 0$) due to higher inter-generational human capital spillovers and higher population growth rate whereas g_A and g_y are increasing in ϵ (that is, $\frac{\partial g_A}{\partial \epsilon} > 0$ and $\frac{\partial g_y}{\partial \epsilon} > 0$) due to higher inter-generational human capital spillovers only when $\theta = \frac{\mu}{\tau\epsilon}$

Intuitively, we know from Lemma 2 that an increase in returns to education triggers a child quantity-quality trade-off at the micro level. The threshold value of quality of schooling, $\frac{\mu}{\tau\epsilon}$,

[19] Detailed derivations of eqs. (74) and (75) are again provided in Appendix G.

is decreasing in the value of ϵ. This implies that, ceteris paribus, this critical threshold value decreases as returns to schooling increase when quality of schooling exceeds the threshold. Therefore, ceteris paribus, parents educate their children and bear lesser number of children in response to an increase in returns to education. Similar to the impact of quality of schooling, this micro level trade-off generates a growth-stimulating effect and a growth-impeding effect at the macro level.

$$\frac{\partial g_H}{\partial \epsilon} = (1+g_h)n\left[\underbrace{\left[\frac{1}{(1-\epsilon)} + log\frac{\epsilon(\tau\theta-\mu)}{1-\epsilon}\right]}_{\text{Growth-stimulating effect}} - \underbrace{\frac{1}{1-\epsilon}}_{\text{Growth-impeding effect}}\right].$$

The growth-stimulating effect overpowers the growth-impeding effect of a change in returns to education when quality of schooling exceeds the threshold as shown by eq. (70). Resultantly, an increase in returns to education yields higher rate of technical progress under both innovation and imitation regimes.

Also, the growth rate of per capita income rises as returns to education increase under both the technology regimes when quality of schooling exceeds the threshold. The intuitive explanantion for this effect is similar as the explanantion for a rise in growth rate of per capita income as quality of schooling improves under the two regimes. The economic growth rate, g_Y, is increasing in ϵ as the growth-stimulating effect overpowers the growth-impeding effect of a change in returns to education. Also, it is known from Lemma 2 that the population growth rate is decreasing in returns to education, ϵ. Therefore, the growth rate of per capita income rises as returns to education increase.

When quality of schooling is strictly lower than the threshold, parents make zero investment in education of their children and focus on having more children. Thus, higher population growth drives economic growth in this case. Additionally, it can be observed from eq. (9) that inter-generational human capital spillovers become more productive and spur growth rate of per capita human capital as returns to education increase whereas when $\theta = \frac{\mu}{\tau\epsilon}$, an increase in returns to education yield higher rate of technical progress and, therefore, economic growth under both the regimes due to higher inter-generational human capital spillovers as fertility rate and therefore, population growth rate does not depend on returns to education in this case.

Similar to the effect of quality of schooling, it can be observed from eqs. (63) and (64) that returns to education raise population growth rate by a lesser proportion as compared to the proportionate rise in growth rate of aggregate output under both the technology regimes. Therefore, growth rate of per capita income rises as returns to education rises.

This completes the characterization of the balanced growth path of our decentralized economy.

5 Discussion

This paper formulates an analytical framework to examine the impact of quality of schooling on technical progress and therefore, economic growth of an economy. Since technical advancements can happen through innovation and technology adoption (imitation), one can observe varying impact on economic growth depending upon whether innovation or technology adoption is driving technical progress. Therefore, we characterize two types of economies. The first is an innovation economy where technological improvements occur by innovating upon the local technology frontier. The second is an imitation economy where technological progress occurs by imitating existing foreign technologies. We examine how the endogenous fertility and education decisions at the household level (triggered by schooling quality) influence human capital accumulation at the aggregate level, which in turn, affect the economic growth under the two technology regimes.

We find that the quality of schooling triggers a child quantity-quality trade-off at the micro level when quality of schooling surpasses an endogenously determined threshold under both the technology regimes. When quality of schooling surpasses the threshold, parents invest in the education of their children and bear lesser number of children. However, parents focus on maximizing fertility and do not educate their children when quality of schooling is less than the threshold. This micro-level trade-off generates two types of effects on economic growth at the macro level - a growth-stimulating effect and a growth-impeding effect. Our results show that the former effect dominates over latter only when the quality of schooling is higher than the threshold, and the economy is on a self-sustaining growth path. Alternatively, when the quality of schooling is less than the threshold, parents do not educate their children and focus, instead on maximizing fertility. Higher fertility rate leads to higher population growth, which propels economic growth rate under both innovation and imitation regimes.

Furthermore, it is advantageous for an economy to innovate upon the local technology frontier instead of imitating from the world technology frontier if the rate of human capital accumulation is higher than the growth rate of world technology frontier in the presence of constant or diminishing returns to R&D sector. Also, a mere surpassing of the threshold level of quality schooling is not sufficient enough for an economy to experience a higher growth rate of per capita output as compared to an economy with quality of schooling lower than the threshold level. Under the two technology regimes, quality of schooling should be high enough such that it leads to high enough investments in education of children, entailing that the growth-stimulating effect dominates the growth-impeding effect of quality of schooling.

This research can be extended in several directions in future. First, it is assumed that quality of schooling is exogenous in our analytical framework. One possible extension can be endogenizing quality of schooling. Quality of schooling can be endogenized by introducing an education sector in which teacher-pupil ratio and teacher quality determine the quality of education system of the economy. It will be interesting to examine how the dynamics of the economy change when an additional education sector is introduced. Introduction of education sector can lead to competition between R&D and education sectors for hiring skilled labor, which may in turn, influence the growth dynamics of the economy. Second, we have focused only on skilled labor. Unskilled labor can also be introduced in the present theoretical structure to determine the impact of quality of schooling on the distribution of income between skilled and unskilled workers in the long-run. Third, there exists a possibility that human capital accumulation can be influenced by technological progress as shown by Bucci (2008). This possibility can be explored in the future by including technological progress in the human capital accumulation function. Fourth, we have characterized innovation-only and imitation-only regimes. Akin to Vandenbussche et al. (2006) and Basu & Mehra (2014), a diversified regime can be introduced where both innovation and imitation activities (with unskilled and skilled labor force) lead to technological improvements.

Appendix A Solution to Household's Optimization Exercise

The utility function is described as follows:
Maximize

$$\begin{aligned} u_t &= \log c_{1,t} + \beta_1 \log c_{2,t+1} + \beta_2 \log(h_{t+1} n_t) \\ &\text{subject to} \\ w_t h_t(1-\tau n_t) &= c_{1,t} + s_t + e_t(w_t h_t) n_t \\ c_{2,t+1} &= (1+r_{t+1}) s_t \\ h_{t+1} &= (\mu + \theta e_t)^{\epsilon} h_t, \quad \epsilon < 1 \end{aligned}$$

After substituting for $c_{2,t+1}$ and h_{t+1}, the lagrangian for this problem is formulated as :

$$\begin{aligned} L = \log c_{1,t} + \beta_1 \log[(1+r_{t+1})s_t] + \beta_2 \log n_t + \beta_2 \epsilon \log(\mu + \theta e_t) + \beta_2 \log h_t \\ + \psi[w_t h_t(1-\tau n_t) - c_{1,t} - s_t - e_t n_t(w_t h_t)] \end{aligned}$$

The choice variables are $c_{1,t}$, s_t,e_t and n_t.The first-order conditions are:

$$\frac{\partial L}{\partial c_{1,t}} = 0 \Leftrightarrow \frac{1}{c_{1,t}} - \psi = 0 \Leftrightarrow c_{1,t} = \frac{1}{\psi}. \tag{A.1}$$

$$\frac{\partial L}{\partial s_t} = 0 \Leftrightarrow \frac{\beta_1}{s_t} - \psi = 0 \Leftrightarrow s_t = \frac{\beta_1}{\psi}. \tag{A.2}$$

$$\frac{\partial L}{\partial n_t} = 0 \Leftrightarrow \frac{\beta_2}{n_t} - \psi\tau w_t h_t - \psi e_t w_t h_t = 0 \Leftrightarrow \frac{\beta_2}{n_t} = \psi[\tau + e_t] w_t h_t \Leftrightarrow n_t = \frac{\beta_2}{\psi[\tau + e_t] w_t h_t}. \tag{A.3}$$

$$\frac{\partial L}{\partial e_t} = 0 \Leftrightarrow \frac{\beta_2 \epsilon\theta}{\mu + \theta e_t} - \psi n_t w_t h_t = 0 \Leftrightarrow n_t = \frac{\beta_2 \epsilon\theta}{\psi[\mu + \theta e_t] w_t h_t}. \tag{A.4}$$

From eqs. (A.3) and (A.4), the left hand side can be equated to yield:

$$\mu + \theta e_t = \epsilon\theta[\tau + e_t] \Leftrightarrow \mu - \epsilon\theta\tau = e_t\theta[\epsilon - 1]$$

$$e_t = \frac{\mu - \epsilon\theta\tau}{\theta(\epsilon - 1)} = \frac{\epsilon\theta\tau - \mu}{\theta(1 - \epsilon)}$$

Hence, we have:

$$e_t = \begin{cases} 0, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon}; \\ \frac{\tau\theta\epsilon - \mu}{\theta(1 - \epsilon)}, & \text{otherwise.} \end{cases} \tag{A.5}$$

Next, we know that the budget constraint is given by:

$$w_t h_t(1 - \tau n_t) = c_{1,t} + s_t + e_t(w_t h_t) n_t.$$

From eq. (A.3), $e_t n_t (w_t h_t)$ can be expressed as:

$$e_t n_t (w_t h_t) = \frac{\beta_2}{\psi} - \tau n_t w_t h_t. \tag{A.6}$$

Substituting from eqs. (A.1), (A.2) and (A.6), the budget constraint can be expressed as:

$$w_t h_t - \tau n_t w_t h_t = \frac{1}{\psi} + \frac{\beta_1}{\psi} + \frac{\beta_2}{\psi} - \tau n_t w_t h_t$$

which on simplifying leads to:

$$\psi = \frac{1 + \beta_1 + \beta_2}{w_t h_t} \tag{A.7}$$

whose substitution into eqs. (A.1) and (A.2) yields:

$$c_{1,t} = \frac{w_t h_t}{1 + \beta_1 + \beta_2};$$
$$s_t = \frac{\beta_1 w_t h_t}{1 + \beta_1 + \beta_2},$$

Substituting for e_t from eq. (A.5) and for ψ from eq. (A.7) in eq. (A.4), yields:

$$n_t = \begin{cases} \frac{\beta_2 \epsilon\theta}{(1 + \beta_1 + \beta_2)\mu}, & \text{if } \theta < \frac{\mu}{\tau\epsilon}; \\ \frac{\beta_2}{(1 + \beta_1 + \beta_2)\tau}, & \text{if } \theta = \frac{\mu}{\tau\epsilon}; \\ \frac{\beta_2 \theta(1 - \epsilon)}{(1 + \beta_1 + \beta_2)(\tau\theta - \mu)}, & \text{otherwise.} \end{cases}$$

This completes the solution to the utility maximization exercise of households.

Appendix B Derivation of Asset market clearing condition

It is known from eq.(33) that final goods market clearing condition is given by:

$$Y_t = c_{1,t}N_t + c_{2,t}N_{t-1} + E_t$$

We know from national accounting that GNP can be calculated using the final use approach or the income approach. Let GNP be denoted by Z_t. From the final use side, it is known that Z_t is used for consumption, investment in R&D activities and for incurring education expenditure on children, i.e.,

$$Z_t = c_{1,t}N_t + c_{2,t}N_{t-1} + E_t + p_t^A(A_{t+1} - A_t) = Y_t + p_t^A(A_{t+1} - A_t). \tag{B.1}$$

From the income side, gross national income (GNI) is given by

$$GNI = w_Y H_t^Y + w_A H_t^A + w_I H_t^I + A_t\pi_t \tag{B.2}$$

Substituting from eqs. (14), (20), (22) and (31), it can be observed that

$$GNI = Y_t + p_t^A(A_{t+1} - A_t) = Z_t$$

Thus, the value of GNI is equivalent to value of GNP. Equating eqs.(B.1) and (B.2), we have

$$\begin{aligned} c_{1,t}N_t + c_{2,t}N_{t-1} + E_t + p_t^A(A_{t+1} - A_t) &= w_Y H_t^Y + w_A H_t^A + w_I H_t^I + A_t\pi_t \\ &= w_t H_t + A_t\pi_t \end{aligned} \tag{B.3}$$

where the last term on the right-hand side is derived using eq.(32) and $w_Y = w_A = w_I = w_t$ at equilibirum. Now, it is known from the household's budget constraint that $w_t h_t(1 - \tau n_t) = c_{1,t} + s_t + e_t(w_t h_t)n_t$. Multiplying both sides of the household's budget constraint by N_t, we get

$$w_t H_t = c_{1,t}N_t + s_t N_t + E_t$$

where $E_t = e_t(w_t h_t)n_t N_t$.Substituting back in eq.(B.3), we deduce

$$p_t^A(A_{t+1} - A_t) = s_t N_t + A_t\pi_t - c_{2,t}N_{t-1} \tag{B.4}$$

It is known from the research arbitrage condition that $A_t\pi_t = (1 + r_t)p_{t-1}^A A_t - p_t^A A_t$.Substituting for $A_t\pi_t$ and using that $c_{2,t}N_{t-1} = (1 + r_t)s_{t-1}N_{t-1}$, we get that

$$p_t^A A_{t+1} = s_t N_t + (1 + r_t)p_{t-1}^A A_t - c_{2,t}N_{t-1} = s_t N_t + (1 + r_t)(p_{t-1}^A A_t - s_{t-1}N_{t-1}). \tag{B.5}$$

Since initial assets are given by $p_{-1}^A A_0 - s_{-1}N_{-1}$at $t = 0$, we get the following asset market clearing condition for any period $t > 0$,

$$p_t^A A_{t+1} = s_t N_t.$$

Appendix C Derivations of Eqs. (56) and (57)

We derive the conditions when an economy with high quality of schooling $\left(\theta > \frac{\mu}{\tau\epsilon}\right)$ exhibits a higher per capita output growth rate as compared to an economy with lower quality of schooling $\left(\theta \leq \frac{\mu}{\tau\epsilon}\right)$. We assume that when $\theta > \frac{\mu}{\tau\epsilon}$, quality of schooling is denoted by θ_h for that particular economy whereas quality of schooling is denoted by θ_l for an economy with quality of schooling less than the threshold $\left(\theta \leq \frac{\mu}{\tau\epsilon}\right)$. An economy with higher schooling quality (θ_h) will grow at a higher rate as compared to an economy with lower quality of schooling (θ_l) when the following condition holds true under both the technology regimes:

$$g_{y,\theta_h} > g_{y,\theta_l} \tag{C.1}$$

We first, derive the condition for innovation economy. At steady state, growth rate of per capita output under innovation regime is given by:

$$g_y = (1 + g_h)^{\frac{1-\phi+\lambda(1-\alpha)}{1-\phi}} n^{\frac{\lambda(1-\alpha)}{1-\phi}} - 1.$$

Thus, we have

$$(1 + g_{h,\theta_h})^{\frac{1-\phi+\lambda(1-\alpha)}{1-\phi}} (n_{\theta_h})^{\frac{\lambda(1-\alpha)}{1-\phi}} > (1 + g_{h,\theta_l})^{\frac{1-\phi+\lambda(1-\alpha)}{1-\phi}} (n_{\theta_l})^{\frac{\lambda(1-\alpha)}{1-\phi}}, \tag{C.2}$$

where g_{h,θ_h} and g_{h,θ_l} are per capita human capital accumulation rates when $\theta > \frac{\mu}{\tau\epsilon}$ and $\theta \leq \frac{\mu}{\tau\epsilon}$ respectively and n_{θ_h} and n_{θ_l} are the fertility rates when $\theta > \frac{\mu}{\tau\epsilon}$ and $\theta \leq \frac{\mu}{\tau\epsilon}$ respectively. Substituting from eqs. (9) and (8) for $(1 + g_{h,\theta_h})$, $(1 + g_{h,\theta_l})$, n_{θ_h} and n_{θ_l} to get:

$$\left[\frac{(\tau\theta_h - \mu)\epsilon}{1-\epsilon}\right]^{\epsilon(1-\phi+\lambda(1-\alpha))} \left[\frac{\theta_h(1-\epsilon)}{(\tau\theta_h - \mu)}\right]^{\lambda(1-\alpha)} > \mu^{\epsilon(1-\phi+\lambda(1-\alpha)} \left[\frac{\epsilon\theta_l}{\mu}\right]^{\lambda(1-\alpha)}$$

which on simplification, yields:

$$\theta_h > \theta_l \left[\frac{\epsilon(\tau\theta_h - \mu)}{\mu(1-\epsilon)}\right]^{\frac{\lambda(1-\alpha)-\epsilon(1-\phi+\lambda(1-\alpha))}{\lambda(1-\alpha)}}. \tag{C.3}$$

Similarly, we derive the condition for imitation economy. Under imitation regime, the growth rate of per capita output given by:

$$g_y = (1 + g_{\bar{A}})^{\frac{1}{2-\phi}} (1 + g_h)^{\frac{2-\phi+\lambda(1-\alpha)}{2-\phi}} n^{\frac{\lambda(1-\alpha)}{2-\phi}} - 1.$$

Thus, we have

$$(1 + g_{\bar{A}})^{\frac{1}{2-\phi}} (1 + g_{h,\theta_h})^{\frac{2-\phi+\lambda(1-\alpha)}{2-\phi}} (n_{\theta_h})^{\frac{\lambda(1-\alpha)}{2-\phi}} > (1 + g_{\bar{A}})^{\frac{1}{2-\phi}} (1 + g_{h,\theta_l})^{\frac{2-\phi+\lambda(1-\alpha)}{1-\phi}} (n_{\theta_l})^{\frac{\lambda(1-\alpha)}{2-\phi}}.$$

Substituting from eqs. (9) and (8) for $(1+g_{h,\theta_h})$, $(1+g_{h,\theta_l})$, n_{θ_h} and n_{θ_l}, we derive:

$$\left[\frac{(\tau\theta_h-\mu)\epsilon}{1-\epsilon}\right]^{\epsilon(2-\phi+\lambda(1-\alpha))}\left[\frac{\theta_h(1-\epsilon)}{(\tau\theta_h-\mu)}\right]^{\lambda(1-\alpha)} > \mu^{\epsilon(2-\phi+\lambda(1-\alpha))}\left[\frac{\epsilon\theta_l}{\mu}\right]^{\lambda(1-\alpha)},$$

which simplifies to:

$$\theta_h > \theta_l\left[\frac{\epsilon(\tau\theta_h-\mu)}{\mu(1-\epsilon)}\right]^{\frac{\lambda(1-\alpha)-\epsilon(2-\phi+\lambda(1-\alpha))}{\lambda(1-\alpha)}}. \tag{C.4}$$

This completes derivations of eqs. (56) and (57). To prove that these two conditions hold true under the two technology regimes, we postulate that,

$$\frac{\lambda(1-\alpha)-\epsilon(1-\phi+\lambda(1-\alpha))}{\lambda(1-\alpha)} > 1, \quad \text{and} \quad \frac{\lambda(1-\alpha)-\epsilon(2-\phi+\lambda(1-\alpha))}{\lambda(1-\alpha)} > 1.$$

which can be simplified to yield the following expressions:

$$\frac{-\epsilon(1-\phi+\lambda(1-\alpha))}{\lambda(1-\alpha)} > 0, \quad \text{and} \quad \frac{-\epsilon(2-\phi+\lambda(1-\alpha))}{\lambda(1-\alpha)} > 0.$$

This is a contradiction as $\phi < 1$, $\lambda < 1$, $\alpha < 1$ and $\epsilon < 1$. Therefore,

$$\frac{\lambda(1-\alpha)-\epsilon(1-\phi+\lambda(1-\alpha))}{\lambda(1-\alpha)} < 1, \quad \text{and} \quad \frac{\lambda(1-\alpha)-\epsilon(2-\phi+\lambda(1-\alpha))}{\lambda(1-\alpha)} < 1. \tag{C.5}$$

Further, we know that $\theta > \dfrac{\mu}{\tau\epsilon}$. Multiplying both sides by τ and then, subtracting μ from both sides yields

$$\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon} > \mu,$$

Since $\mu \geq 1$, we have:

$$\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon} > 1. \tag{C.6}$$

Thus, $\left[\dfrac{(\tau\theta_h-\mu)\epsilon}{\mu(1-\epsilon)}\right] > 1$. Therefore, eqs. (C.5) and (C.6) together imply that

$$\left[\frac{(\tau\theta_h-\mu)\epsilon}{\mu(1-\epsilon)}\right]^{\frac{\lambda(1-\alpha)-\epsilon(1-\phi+\lambda(1-\alpha))}{\lambda(1-\alpha)}} > 1, \quad \text{and} \quad \left[\frac{(\tau\theta_h-\mu)\epsilon}{\mu(1-\epsilon)}\right]^{\frac{\lambda(1-\alpha)-\epsilon(2-\phi+\lambda(1-\alpha))}{\lambda(1-\alpha)}} > 1. \tag{C.7}$$

Appendix D Derivations of Eq. (60)

We know that $(1+g_H) = (1+g_h).n$. Differentiating both the sides w.r.t θ yields:

$$\frac{\partial g_H}{\partial\theta} = (1+g_h)\frac{\partial n}{\partial\theta} + n\frac{\partial g_h}{\partial\theta}. \tag{D.1}$$

When $\theta > \dfrac{\mu}{\tau\epsilon}$, we know from Lemma 1 that $\frac{\partial n_t}{\partial\theta} = -\frac{\mu\beta_2(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2}$ and it is given that $(1+g_h) = \left[\dfrac{\epsilon(\tau\theta-\mu)}{(1-\epsilon)}\right]^{\epsilon}$ from eq (9). Differentiating g_h w.r.t θ, we get that:

$$\frac{\partial g_h}{\partial\theta} = \left[\frac{\epsilon(\tau\theta-\epsilon)}{1-\epsilon}\right]^{\epsilon} . \frac{\epsilon\tau}{\tau\theta-\mu} = \frac{(1+g_h)\epsilon\tau}{\tau\theta-\mu}. \tag{D.2}$$

Substituting this into eq. (D.1), we get that:

$$\frac{\partial g_H}{\partial\theta} = (1+g_h)\frac{-\mu\beta_2(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2} + (1+g_h)\frac{\epsilon\tau}{\tau\theta-\mu} * n,$$

Substituting for n from eq. (8),

$$\begin{aligned}\frac{\partial g_H}{\partial\theta} &= (1+g_h)\left[\frac{\epsilon\tau\beta_2\theta(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2} - \frac{\mu\beta_2(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2}\right] \\ &= (1+g_h)[\epsilon\tau\theta-\mu]\frac{\beta_2(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2}.\end{aligned}$$

This completes derivation of eq. (60).

Appendix E Derivations of Eqs. (65), (66), (67) and (68)

The growth rate of per capita income can be expressed as:

$$(1+g_y) = \frac{(1+g_Y)}{n}. \tag{E.1}$$

Under innovation regime, substituting for $(1+g_Y)$ from eq. (51) and simplifying, we get:

$$(1+g_y) = (1+g_h)^{\frac{1-\phi+\lambda(1-\alpha)}{1-\phi}} n^{\frac{\lambda(1-\alpha)}{1-\phi}}. \tag{E.2}$$

Taking log on both sides and differentiating w.r.t θ, yields,

$$\frac{1}{1+g_y}\frac{\partial g_y}{\partial\theta} = \frac{1-\phi+\lambda(1-\alpha)}{(1+g_h)1-\phi}\frac{\partial g_h}{\partial\theta} + \frac{\lambda(1-\alpha)}{(1-\phi)n}\frac{\partial n}{\partial\theta}, \tag{E.3}$$

When $\theta > \dfrac{\mu}{\tau\epsilon}$, substituting for n from eq. (8), $\dfrac{\partial n}{\partial\theta}$ from Lemma 1 and $\dfrac{\partial g_h}{\partial\theta}$ from eq. (D.2) yields:

$$\frac{\partial g_y}{\partial\theta} = \frac{1+g_y}{1-\phi}\left[\frac{(1-\phi+\lambda(1-\alpha))\tau\epsilon}{\tau\theta-\mu} - \frac{\mu\lambda(1-\alpha)}{\theta(\tau\theta-\mu)}\right].$$

It can be observed that $\dfrac{\partial g_y}{\partial\theta} > 0$, if

$$\frac{(1-\phi+\lambda(1-\alpha))\tau\epsilon}{(\tau\theta-\mu)} > \frac{\mu\lambda(1-\alpha)}{\theta(\tau\theta-\mu)} \Leftrightarrow \theta > \frac{\mu\lambda(1-\alpha)}{\tau\epsilon(1-\phi+\lambda(1-\alpha))}.$$

This holds true as $\theta > \dfrac{\mu}{\tau\epsilon}$, $\lambda < 1$ and $\phi < 1$. Thus, $\frac{\partial g_y}{\partial\theta} > 0$.

Next, we consider the case when $\theta < \frac{\mu}{\tau\epsilon}$. Substituting for n from eq. (8), $\dfrac{\partial g_h}{\partial\theta}$ from eq.(61) and $\dfrac{\partial n}{\partial\theta}$ from Lemma 1 in eq. (E.3), we deduce:

$$\frac{\partial g_y}{\partial\theta} = \left[\frac{\lambda(1-\alpha)(1+g_y)}{\theta(1-\phi)}\right].$$

We now, consider the imitation regime. Substituting for $(1+g_Y)$ from eq. (52) and simplifying, we get:

$$(1+g_y) = (1+g_{\bar{A}})^{\frac{1}{2-\phi}}(1+g_h)^{\frac{2-\phi+\lambda(1-\alpha)}{2-\phi}}n^{\frac{\lambda(1-\alpha)}{2-\phi}}. \tag{E.4}$$

Taking log on both sides and differentiating w.r.t θ, we get:

$$\frac{1}{1+g_y}\frac{\partial g_y}{\partial\theta} = \frac{2-\phi+\lambda(1-\alpha)}{2-\phi(1+g_h)}\frac{\partial g_h}{\partial\theta} + \frac{\lambda(1-\alpha)}{(2-\phi)n}\frac{\partial n}{\partial\theta}. \tag{E.5}$$

When $\theta > \frac{\mu}{\tau\epsilon}$, substituting for n from eq. (8), $\dfrac{\partial n}{\partial\theta}$ from Lemma 1 and $\dfrac{\partial g_h}{\partial\theta}$ from eq. (D.2), we have:

$$\frac{\partial g_y}{\partial\theta} = \frac{1+g_y}{2-\phi}\left[\frac{(2-\phi+\lambda(1-\alpha))\tau\epsilon}{\tau\theta-\mu} - \frac{\mu\lambda(1-\alpha)}{\theta(\tau\theta-\mu)}\right].$$

Now, $\dfrac{\partial g_y}{\partial\theta} > 0$ if

$$\frac{(2-\phi+\lambda(1-\alpha))\tau\epsilon}{\tau\theta-\mu} > \frac{\mu\lambda(1-\alpha)}{\theta(\tau\theta-\mu)} \Leftrightarrow \theta > \frac{\mu\lambda(1-\alpha)}{\tau\epsilon(2-\phi+\lambda(1-\alpha))}.$$

This also holds true as $\theta > \dfrac{\mu}{\tau\epsilon}$, $\lambda < 1$ and $\phi < 2$. Therefore, $\frac{\partial g_y}{\partial\theta} > 0$.

We next, consider the case when $\theta < \frac{\mu}{\tau\epsilon}$. When $\theta < \frac{\mu}{\tau\epsilon}$, substituting for n from eq. (8), $\dfrac{\partial g_h}{\partial\theta}$ from eq.(61) and $\dfrac{\partial n}{\partial\theta}$ from Lemma 1 in eq. (E.5) yields:

$$\frac{\partial g_y}{\partial\theta} = \left[\frac{\lambda(1-\alpha)(1+g_y)}{\theta(2-\phi)}\right].$$

This completes derivations of eqs. (65), (66), (67) and (68).

Appendix F Derivations of Eqs. (70) and (71)

We know that:$(1+g_H) = (1+g_h).n$. Differentiating both the sides w.r.t ϵ, we get that:

$$\frac{\partial g_H}{\partial\epsilon} = (1+g_h)\frac{\partial n}{\partial\epsilon} + n\frac{\partial g_h}{\partial\epsilon} \tag{F.1}$$

When $\theta > \frac{\mu}{\tau\epsilon}$, we know from the interior solution of eq. (9) that $(1+g_h) = \left[\frac{\epsilon(\tau\theta-\mu)}{(1-\epsilon)}\right]^{\epsilon}$. Taking log on both sides and differentiating w.r.t ϵ, we get the following expression:

$$\begin{aligned}\frac{1}{1+g_h}\frac{\partial g_h}{\partial \epsilon} &= 1 + log\epsilon + log(\tau\theta-\mu) + \frac{\epsilon}{1-\epsilon} - log(1-\epsilon) \\ &= (1+g_h)\left[\frac{1}{(1-\epsilon)} + log\left[\frac{\epsilon(\tau\theta-\mu)}{1-\epsilon}\right]\right]. \end{aligned} \tag{F.2}$$

Also, from Lemma 2, we have $\frac{\partial n_t}{\partial \epsilon} = \frac{-\beta_2\theta}{(1+\beta_1+\beta_2)(\tau\theta-\mu)}$. Substituting for $\frac{\partial n}{\partial \epsilon}$ from Lemma 2 and $\frac{\partial g_h}{\partial \epsilon}$ from eq. (F.2) into eq. (F.1), we derive that:

$$\frac{\partial g_H}{\partial \epsilon} = \frac{-\beta_2\theta(1+g_h)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)} + (1+g_h)n\left[\frac{1}{(1-\epsilon)} + log\frac{\epsilon(\tau\theta-\mu)}{1-\epsilon}\right]$$

Substituting for $\frac{\beta_2\theta}{(1+\beta_1+\beta_2)(\tau\theta-\mu)}$ from eq. (8), yields:

$$= (1+g_h)\left[n\left[\frac{1}{(1-\epsilon)} + log\frac{\epsilon(\tau\theta-\mu)}{1-\epsilon}\right] - \frac{n}{1-\epsilon}\right] = (1+g_H)log\left[\frac{\epsilon(\tau\theta-\mu)}{1-\epsilon}\right].$$

We next derive the expression for $\frac{\partial g_H}{\partial \epsilon}$ when $\theta \leq \frac{\mu}{\tau\epsilon}$. When $\theta \leq \frac{\mu}{\tau\epsilon}$, it is known from eq. (9) that:$(1+g_h) = \mu^{\epsilon}$. Taking log on both sides and differentiating g_h w.r.t ϵ yields:

$$\frac{\partial g_h}{\partial \epsilon} = (1+g_h)log\mu. \tag{F.3}$$

Substituting for $\frac{\partial n}{\partial \epsilon}$ and $\frac{\partial g_h}{\partial \epsilon}$ from Lemma 2 and eq. (F.3) into eq. (F.1), we deduce that:

$$\frac{\partial g_H}{\partial \epsilon} = \begin{cases}(1+g_H)\left[\frac{1}{\epsilon} + log\mu\right] > 0, & \text{if } \theta < \frac{\mu}{\tau\epsilon}, \\ (1+g_H)\left[log\mu\right] > 0, & \theta = \frac{\mu}{\tau\epsilon}.\end{cases} \tag{F.4}$$

This completes the derivations of eqs. (70) and (71).

Appendix G Derivations of Eqs. (72), (73), (74) and (75)

It is known from eq. (E.2) that the growth rate of per capita income under innovation regime is given by:

$$(1+g_y) = (1+g_h)^{\frac{1-\phi+\lambda(1-\alpha)}{1-\phi}} n^{\frac{\lambda(1-\alpha)}{1-\phi}}. \tag{G.1}$$

Taking log on both sides and differentiating w.r.t ϵ yields:

$$\frac{1}{1+g_y}\frac{\partial g_y}{\partial \epsilon} = \frac{1-\phi+\lambda(1-\alpha)}{(1+g_h)1-\phi}\frac{\partial g_h}{\partial \epsilon} + \frac{\lambda(1-\alpha)}{(1-\phi)n}\frac{\partial n}{\partial \epsilon}. \tag{G.2}$$

When $\theta > \frac{\mu}{\tau\epsilon}$, substituting for n from eq. (8), $\frac{\partial n}{\partial \epsilon}$ from Lemma 2 and $\frac{1}{1+g_h}\frac{\partial g_h}{\partial \epsilon}$ from eq. (F.2), we get:

$$\frac{\partial g_y}{\partial \epsilon} = \frac{(1-\phi+\lambda(1-\alpha))(1+g_y)}{1-\phi}\left[\frac{1}{1-\epsilon} + log\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon}\right] - \frac{(1+g_y)\lambda(1-\alpha)}{(1-\epsilon)(1-\phi)}. \tag{G.3}$$

Now, it can be observed that $\frac{\partial g_y}{\partial \epsilon} > 0$, if

$$(1-\phi+\lambda(1-\alpha))\left[\frac{1}{1-\epsilon} + log\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon}\right] > \frac{\lambda(1-\alpha)}{1-\epsilon},$$

which simplifies to

$$\frac{(1-\phi)}{\lambda(1-\alpha)} + \frac{(1-\epsilon)(1-\phi+\lambda(1-\alpha))}{\lambda(1-\alpha)}log\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon} > 0. \tag{G.4}$$

We know that $\theta > \frac{\mu}{\tau\epsilon}$, $\epsilon < 1$, $\lambda < 1$, $\alpha < 1$ and $\phi < 1$. Also, $\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon} > 1$ from eq. (C.6). Therefore, $\frac{\partial g_y}{\partial \epsilon} > 0$.

We, next, consider the case where $\theta \leq \frac{\mu}{\tau\epsilon}$. Substituting for n from eq. (8), $\frac{\partial g_h}{\partial \epsilon}$ from eq. (F.3) and $\frac{\partial n}{\partial \epsilon}$ from Lemma 2 in eq. (G.2) yields:

$$\frac{\partial g_y}{\partial \epsilon} = \begin{cases} (1+g_y)\left[1+\frac{\lambda(1-\alpha)}{1-\phi}\right]log\mu + \frac{\lambda(1-\alpha)(1+g_y)}{(1-\phi)\epsilon} > 0; & \text{if } \theta < \frac{\mu}{\tau\epsilon}, \\ (1+g_y)\left[1+\frac{\lambda(1-\alpha)}{1-\phi}\right]log\mu > 0, & \theta = \frac{\mu}{\tau\epsilon} \text{ and} \end{cases} \tag{G.5}$$

We, next, derive the expression for $\frac{\partial g_y}{\partial \epsilon}$ under imitation regime. We know from eq. (64) that:

$$log(1+g_y) = \frac{1}{(2-\phi)}log(1+g_{\bar{A}}) + \frac{2-\phi+\lambda(1-\alpha)}{(2-\phi)}log(1+g_h) + \frac{\lambda(1-\alpha)}{2-\phi}logn,$$

Differentiating w.r.t ϵ, we get:

$$\frac{1}{1+g_y}\frac{\partial g_y}{\partial \epsilon} = \frac{2-\phi+\lambda(1-\alpha)}{2-\phi(1+g_h)}\frac{\partial g_h}{\partial \epsilon} + \frac{\lambda(1-\alpha)}{(2-\phi)n}\frac{\partial n}{\partial \epsilon}. \tag{G.6}$$

When $\theta > \frac{\mu}{\tau\epsilon}$, substituting for n from eq. (8), $\frac{\partial n}{\partial \epsilon}$ from Lemma 2 and $\frac{\partial g_h}{\partial \epsilon}$ from eq. (F.2) yields:

$$\frac{\partial g_y}{\partial \epsilon} = \frac{(2-\phi+\lambda(1-\alpha))(1+g_y)}{2-\phi}\left[\frac{1}{1-\epsilon} + log\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon}\right] - \frac{(1+g_y)\lambda(1-\alpha)}{(1-\epsilon)(2-\phi)}.$$

Now, $\frac{\partial g_y}{\partial \epsilon} > 0$, if

$$(2-\phi+\lambda(1-\alpha))\left[\frac{1}{1-\epsilon} + log\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon}\right] > \frac{\lambda(1-\alpha)}{1-\epsilon}, \tag{G.7}$$

which simplifies to

$$\frac{(2-\phi)}{\lambda(1-\alpha)} + \frac{(1-\epsilon)(2-\phi+\lambda(1-\alpha))}{\lambda(1-\alpha)} log \frac{(\tau\theta-\mu)\epsilon}{1-\epsilon} > 0. \tag{G.8}$$

We know that $\theta > \frac{\mu}{\tau\epsilon}$, $\epsilon < 1$, $\lambda < 1$, $\alpha < 1$ and $\phi < 1$. Also, $\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon} > 1$ from eq. (C.6). Therefore, $\frac{\partial g_y}{\partial \epsilon} > 0$. Alternatively, when $\theta \leq \frac{\mu}{\tau\epsilon}$, substituting for n from eq. (8), $\frac{\partial g_h}{\partial \epsilon}$ from eq. (F.3) and $\frac{\partial n}{\partial \epsilon}$ from Lemma 2 in eq. (G.6) yields:

$$\frac{\partial g_y}{\partial \epsilon} = \begin{cases} (1+g_y)\left[1+\frac{\lambda(1-\alpha)}{2-\phi}\right] log\mu + \frac{\lambda(1-\alpha)(1+g_y)}{(2-\phi)\epsilon} > 0; & \text{if } \theta < \frac{\mu}{\tau\epsilon}, \\ (1+g_y)\left[1+\frac{\lambda(1-\alpha)}{2-\phi}\right] log\mu > 0, & \theta = \frac{\mu}{\tau\epsilon}. \end{cases} \tag{G.9}$$

This completes derivations of eqs. (72), (73), (74) and (75).

References

Aghion, P. & Howitt, P. (1992), ‘A model of growth through creative destruction’, *Econometrica* **60**(2).

Altinok, N. & Aydemir, A. (2017), ‘Does one size fit all? the impact of cognitive skills on economic growth’, *Journal of Macroeconomics* **53**, 176–190.

Andreoni, J. (1989), ‘Giving with impure altruism: Applications to charity and ricardian equivalence’, *Journal of Political Economy* **97**(6), 1447–1458.

Ang, J. B., Madsen, J. B. & Islam, M. R. (2011), ‘The effects of human capital composition on technological convergence’, *Journal of Macroeconomics* **33**(3), 465–476.

Annual Status of Education Report (ASER) (2019), ‘Early years’. Retrieved from `http://img.asercentre.org/docs/aserreport2019earlyyearsfinal.pdff` [Accessed: 7 February, 2020].

Baldanzi, A., Bucci, A. & Prettner, K. (2021), ‘Children’s health, human capital accumulation, and r&d-based economic growth’, *Macroeconomic Dynamics* **25**(3), 651–668.

Barro, R. J. & Lee, J. W. (2013), ‘A new data set of educational attainment in the world, 1950–2010’, *Journal of Development Economics* **104**, 184–198.

Basu, S. & Mehra, M. K. (2014), ‘Endogenous human capital formation, distance to frontier and growth’, *Research in Economics* **68**(2), 117–132.

Becker, G. S. (1960), An economic analysis of fertility, *in* ‘Demographic and economic change in developed countries’, Columbia University Press, pp. 209–240.

Benhabib, J., Perla, J. & Tonetti, C. (2014), ‘Catch-up and fall-back through innovation and imitation’, *Journal of Economic Growth* **19**, 1–35.

Benhabib, J., Perla, J. & Tonetti, C. (2021), ‘Reconciling models of diffusion and innovation: A theory of the productivity distribution and technology frontier’, *Econometrica* **89**(5), 2261–2301.

Benhabib, J. & Spiegel, M. M. (1994), 'The role of human capital in economic development evidence from aggregate cross-country data', *Journal of Monetary economics* **34**(2), 143–173.

Boikos, S., Bucci, A. & Stengos, T. (2013), 'Non-monotonicity of fertility in human capital accumulation and economic growth', *Journal of Macroeconomics* **38**, 44–59.

Boikos, S., Bucci, A. & Stengos, T. (2022), 'Leisure and innovation in horizontal r&d-based growth', *Economic Modelling* **107**, 105730.

Boucekkine, R., Croix, D. d. l. & Licandro, O. (2003), 'Early mortality declines at the dawn of modern growth', *The Scandinavian Journal of Economics* **105**(3), 401–418.

Boucekkine, R., De la Croix, D. & Licandro, O. (2002), 'Vintage human capital, demographic trends, and endogenous growth', *Journal of Economic Theory* **104**(2), 340–375.

Brander, J. A. & Dowrick, S. (1994), 'The role of fertility and population in economic growth', *Journal of Population Economics* **7**(1), 1–25.

Bucci, A. (2008), 'Population growth in a model of economic growth with human capital accumulation and horizontal r&d', *Journal of Macroeconomics* **30**(3), 1124–1147.

Bucci, A., Prettner, K. et al. (2020), 'Endogenous education and the reversal in the relationship between fertility and economic growth', *Journal of Population Economics* **33**(3), 1025–1068.

Campbell, S. G. & Üngör, M. (2020), 'Revisiting human capital and aggregate income differences', *Economic Modelling* .

Caselli, F. & Coleman, W. J. (2001), 'Cross-country technology diffusion: The case of computers', *American Economic Review* **91**(2), 328–335.

Caselli, F. & Wilson, D. J. (2004), 'Importing technology', *Journal of monetary Economics* **51**(1), 1–32.

Castelló-Climent, A. & Hidalgo-Cabrillana, A. (2012), 'The role of educational quality and quantity in the process of economic development', *Economics of Education Review* **31**(4), 391–409.

Cervellati, M., Meyerheim, G. & Sunde, U. (2023), 'The empirics of economic growth over time and across nations: a unified growth perspective', *Journal of Economic Growth* **28**(2), 173–224.

Ciccone, A. & Papaioannou, E. (2009), 'Human capital, the structure of production, and growth', *The Review of Economics and Statistics* **91**(1), 66–82.

Comin, D. & Hobijn, B. (2004), 'Cross-country technology adoption: making the theories face the facts', *Journal of monetary Economics* **51**(1), 39–83.

Comin, D. & Hobijn, B. (2010), 'An exploration of technology diffusion', *American economic review* **100**(5), 2031–2059.

Comin, D. & Mestieri, M. (2018), 'If technology has arrived everywhere, why has income diverged?', *American Economic Journal: Macroeconomics* **10**(3), 137–178.

Das, J., Dercon, S., Habyarimana, J. & Krishnan, P. (2004), ‘When can school inputs improve test scores?’, *World Bank Policy Research Working Paper* (3217).

De la Croix, D. & Doepke, M. (2004), ‘Public versus private education when differential fertility matters’, *Journal of Development Economics* **73**(2), 607–629.

Diamond, P. A. (1965), ‘National debt in a neoclassical growth model’, *The American Economic Review* **55**(5), 1126–1150.

Eaton, J. & Kortum, S. (2001), ‘Trade in capital goods’, *European Economic Review* **45**(7), 1195–1235.

Eckstein, Z. & Zilcha, I. (1994), ‘The effects of compulsory schooling on growth, income distribution and welfare’, *Journal of Public Economics* **54**(3), 339–359.

Foster, A. D. & Rosenzweig, M. R. (1996), ‘Technical change and human-capital returns and investments: evidence from the green revolution’, *The American Economic Review* pp. 931–953.

Foster, A. D. & Rosenzweig, M. R. (2010), ‘Microeconomics of technology adoption’, *Annu. Rev. Econ.* **2**(1), 395–424.

Futagami, K. & Konishi, K. (2019), ‘Rising longevity, fertility dynamics, and r& d-based growth’, *Journal of Population Economics* **32**(2), 591–620.

Galor, O. & Weil, D. N. (1996), ‘The gender gap, fertility, and growth’, *The American Economic Review* pp. 374–387.

Galor, O. & Weil, D. N. (2000), ‘Population, technology, and growth: From malthusian stagnation to the demographic transition and beyond’, *American Economic Review* **90**(4), 806–828.

Gilpin, G. & Kaganovich, M. (2012), ‘The quantity and quality of teachers: Dynamics of the trade-off’, *Journal of Public Economics* **96**(3-4), 417–429.

Glewwe, P., Ilias, N. & Kremer, M. (2010), ‘Teacher incentives’, *American Economic Journal: Applied Economics* **2**(3), 205–27.

Gort, M. & Klepper, S. (1982), ‘Time paths in the diffusion of product innovations’, *The economic journal* **92**(367), 630–653.

Griliches, Z. (1957), ‘Hybrid corn: An exploration in the economics of technological change’, *Econometrica, Journal of the Econometric Society* pp. 501–522.

Guilló, M. D., Papageorgiou, C. & Perez-Sebastian, F. (2011), ‘A unified theory of structural change’, *Journal of Economic Dynamics and Control* **35**(9), 1393–1404.

Hanushek, E. A. & Kimko, D. D. (2000), ‘Schooling, labor-force quality, and the growth of nations’, *American Economic Review* **90**(5), 1184–1208.

Hanushek, E. A., Lavy, V. & Hitomi, K. (2008), ‘Do students care about school quality? determinants of dropout behavior in developing countries’, *Journal of Human Capital* **2**(1), 69–105.

Hanushek, E. A. & Woessmann, L. (2012), ‘Do better schools lead to more growth? cognitive skills, economic outcomes, and causation’, *Journal of Economic Growth* **17**(4), 267–321.

Hashimoto, K.-i. & Tabata, K. (2016), ‘Demographic change, human capital accumulation and r&d-based growth’, *Canadian Journal of Economics/Revue canadienne d’économique* **49**(2), 707–737.

Herzer, D., Strulik, H. & Vollmer, S. (2012), ‘The long-run determinants of fertility: one century of demographic change 1900–1999’, *Journal of Economic Growth* **17**(4), 357–385.

Islam, R., Ang, J. B. & Madsen, J. B. (2014), ‘Quality-adjusted human capital and productivity growth’, *Economic Inquiry* **52**(2), 757–777.

Jones, C. I. (1995), ‘R & d-based models of economic growth’, *Journal of Political Economy* **103**(4), 759–784.

Jovanovic, B. & MacDonald, G. M. (1994), ‘Competitive diffusion’, *Journal of Political Economy* **102**(1), 24–52.

Jovanovic, B. & Rob, R. (1989), ‘The growth and diffusion of knowledge’, *The Review of Economic Studies* **56**(4), 569–582.

Kalemli-Ozcan, S. (2002), ‘Does the mortality decline promote economic growth?’, *Journal of Economic Growth* **7**(4), 411–439.

Kalemli-Ozcan, S. (2003), ‘A stochastic model of mortality, fertility, and human capital investment’, *Journal of Development Economics* **70**(1), 103–118.

Kelley, A. C. & Schmidt, R. M. (1994), *Population and income change: Recent evidence*, The World Bank.

Kortum, S. S. (1997), ‘Research, patenting, and technological change’, *Econometrica: Journal of the Econometric Society* pp. 1389–1419.

Krueger, A. B. & Lindahl, M. (2001), ‘Education for growth: Why and for whom?’, *Journal of Economic Literature* **39**(4), 1101–1136.

Li, H. & Zhang, J. (2007), ‘Do high birth rates hamper economic growth?’, *The Review of Economics and Statistics* **89**(1), 110–117.

Lucas Jr, R. E. (1988), ‘On the mechanics of economic development’, *Journal of Monetary Economics* **22**(1), 3–42.

Lucas Jr, R. E. (2009), ‘Ideas and growth’, *Economica* **76**(301), 1–19.

Lucas Jr, R. E. (2015), ‘Human capital and growth’, *American economic review* **105**(5), 85–88.

Lucas Jr, R. E. & Moll, B. (2014), ‘Knowledge growth and the allocation of time’, *Journal of Political Economy* **122**(1), 1–51.

Mankiw, N. G., Romer, D. & Weil, D. N. (1992), ‘A contribution to the empirics of economic growth’, *The Quarterly Journal of Economics* **107**(2), 407–437.

Mansfield, E. (1961), ‘Technical change and the rate of imitation’, *Econometrica: Journal of the Econometric Society* pp. 741–766.

Manuelli, R. E. & Seshadri, A. (2014), ‘Frictionless technology diffusion: The case of tractors’, *American Economic Review* **104**(4), 1368–1391.

Nelson, R. R. & Phelps, E. S. (1966), 'Investment in humans, technological diffusion, and economic growth', *The American Economic Review* **56**(1/2), 69–75.

Neycheva, M. (2016), 'Secondary versus higher education for growth: the case of three countries with different human capital's structure and quality', *Quality & Quantity* **50**(6), 2367–2393.

Papageorgiou, C. & Perez-Sebastian, F. (2006), 'Dynamics in a non-scale r&d growth model with human capital: Explaining the japanese and south korean development experiences', *Journal of Economic Dynamics and Control* **30**(6), 901–930.

Perla, J. & Tonetti, C. (2014), 'Equilibrium imitation and growth', *Journal of Political Economy* **122**(1), 52–76.

Prettner, K., Bloom, D. E. & Strulik, H. (2013), 'Declining fertility and economic well-being: Do education and health ride to the rescue?', *Labour economics* **22**, 70–79.

Rebelo, S. (1991), 'Long-run policy analysis and long-run growth', *Journal of Political Economy* **99**(3), 500–521.

Reimers, F. (1993), 'Time and opportunity to learn in pakistan's schools: Some lessons on the links between research and policy', *Comparative Education* **29**(2), 201–212.

Romer, P. M. (1990), 'Endogenous technological change', *Journal of Political Economy* **98**(5, Part 2), S71–S102.

Soares, R. R. (2005), 'Mortality reductions, educational attainment, and fertility choice', *American Economic Review* **95**(3), 580–601.

Stokey, N. L. (2021), 'Technology and skill: Twin engines of growth', *Review of Economic Dynamics* **40**, 12–43.

Strulik, H. (2005), 'The role of human capital and population growth in r&d-based models of economic growth', *Review of international Economics* **13**(1), 129–145.

Strulik, H., Prettner, K. & Prskawetz, A. (2013), 'The past and future of knowledge-based growth', *Journal of Economic Growth* **18**(4), 411–437.

Tamura, R. (2001), 'Teachers, growth, and convergence', *Journal of Political Economy* **109**(5), 1021–1059.

United Nations, Department of Economic and Social Affairs, Population Division (2019), 'World population prospects 2019 : Highlights'. Retrieved from`https://population.un.org/wpp/Publications/Files/WPP2019Highlights.pdf` [Accessed: 2 August, 2019].

Vandenbussche, J., Aghion, P. & Meghir, C. (2006), 'Growth, distance to frontier and composition of human capital', *Journal of Economic Growth* **11**(2), 97–127.