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Résumé de l'article

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On 'Rusting' Money: Silvio Gesell's Schwundgeld Reconsidered

Part I: The Short Run

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Silvio Gesell hypothesized that money depreciation is economically and socially beneficial, an idea that have often been contended. Here I analyze the spirit of his claims in a Sidrauski model in which households additionally have a 'love of wealth'motive. The analysis is presented in two parts, one focusing on the short and the other one on the long run. In the first part of this work, these features provide micro-foundations for analyzing Gesell's claim in the short run. Contrary to some claims it is shown Gesell's conjectures may indeed be valid in a demand-determined, short-run equilibrium and why money depreciation overcomes the zero lower bound on nominal interest rates. These results are checked against the recent demonetization episode in India and essentially found to be true. Hence, Gesell's hypotheses can be verified for a plausible, short-run environment and may be relevant for current economic, especially, monetary policies.

Keywords: Economic Performance, Depreciating Money, Zero Lower Bound, Demonetization, Love of Wealth

JEL Classifications: E1, E5, O4

[†] Professor Rehme passed away last year. The paper is published as it was originally submitted, subject to conversion to the *Review*'s format..

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1 Introduction

"Money is the football of economic life."[1]

In his main piece of work "The Natural Economic Order" Silvio Gesell, a German merchant and intellectual, developed various insightful arguments to improve the workings of an economy. It was first published in Bern in 1916 and received praise from economists such as Keynes (1936) in his "General Theory of Employment, Money and Interest", ch. 23, and Fisher (1932) in his "Booms and Depressions".

In this paper, I reconsider his idea of *Schwundgeld* (demurrage) and its consequences on economic performance. I analyze whether the spirit of his key conjectures can be justified in a relatively parsimonious, modern theoretical framework. One reason is that Gesell's claims have often been contended by arguing that they cannot be corroborated by 'state-of-the-art' theory.

Although Gesell (1920), p. 78, acknowledges that money is "the football of economic life"and thus (probably) being a key driver of, and essential for, any modern economy, he cautions us by arguing;"Only money that goes out of date like a newspaper, rots like potatoes, rusts like iron, evaporates like ether, is capable of standing the test as an instrument for the exchange of potatoes, newspapers, iron and ether. For such money is not preferred to goods either by the purchaser or the seller. We then part with our goods for money only because we need the money as a means of exchange, not because we expect an advantage from possession of the money." (p. 121)

According to him placing money and commodities on equal 'physical' footing requires that money depreciates, just as normal goods do due to the wear and tear in usage or storage. In particular, he argued the face value of (paper) money depreciates at a certain percentage over a particular period. To regain the previous face value of the money (note) used, people would have to buy stamps to make up for the depreciation the monetary authority would decree for the money note.[2]

[1] Silvio Gesell (1920) The Natural Economic Order.

[2] Consider his example for the American currency: "This $100 note (bill) is shown as it will appear during the week August 4th-11th, thirty-one ten-cent stamps ($3.10) having been attached to it by its various holders on the dated spaces provided for the purpose, one stamp for each week since the beginning of the year. In the course of the year 52 ten-cent stamps ($5.20) must be attached to the $100 note, or in other words it depreciates 5.2% annually at the expense of its holders." Gesell (1920), p. 121/2.

The introduction of such a monetary arrangement would then influence the economy in ways about which he formed, among others, the following four hypotheses.[3]

Gesell Conjecture 1 (GC1) *The introduction of, and, when present, an increase in, the money depreciation rate leads to a higher velocity of money in circulation.*

> "Everyone, of course, tries to avoid the expense of stamping the notes by passing them on - by purchasing something, by paying debts, by engaging labour, or by depositing the notes in the bank, which must at once find borrowers for the money, if necessary by reducing the rate of interest on its loans. In this way, the circulation of money is subjected to pressure." Gesell (1920), p. 123.

Gesell Conjecture 2 (GC2) *Money depreciation coupled with expansionary monetary policy stimulates aggregate demand and through that output and employment.*

> "In all conceivable circumstances, in fair weather and foul, demand will then exactly equal: - The quantity of money circulated and controlled by the State. Multiplied by: The maximum velocity of circulation possible with the existing commercial organization. What is the effect on economic life? The effect is that we now dominate the fluctuations of the market; that the Currency Office, by issuing and withdrawing money, can tune demand to the needs of the market; that demand is no longer controlled by the holders of money, by the fears of the middle classes, the gambling of speculators or the tone of the Stock Exchange, but that its amount is determined absolutely by the Currency Office. The Currency Office now creates demand, just as the State manufactures postage stamps, or as the workers create supply." Gesell (1920), p. 127.

Gesell Conjecture 3 (GC3) *A money depreciation rate is welfare enhancing.*

> "The elimination of interest is the natural result of the natural order of things when undisturbed by artificial interference. Everything in the nature of men as in the nature of economic life urges the continual increase of so-called real capital - an increase which continues even after the complete disappearance of interest. The sole disturber of the peace in this natural order we have shown to be the traditional medium of exchange. The unique and characteristic advantages of this medium of exchange permit the arbitrary postponement of demand, without direct loss to its possessor; whereas supply, on account of the physical characteristics of the wares, punishes delay with losses of all

[3] More elaborate justifications for Gesell's claims and ideas can be found in the working paper version of this paper; see Rehme (2018), appendix F, and the quotes presented at the end of this paper.

> kinds. In defense of their economic welfare, both the individual and the community have been and are at enmity with interest, and they would long ago have eliminated interest if their power had not been trammeled by money." Gesell (1920), p. 190.

Gesell Conjecture 4 (GC4) *A money depreciation rate benefits workers relatively more than capital owners.*

> "By the laws of free competition, the manufacturer's profit must be reduced to the level of a technician's salary - an unpleasant result for many manufacturers whose success was mainly due to their commercial ability. With free money, creative power has become unnecessary in commerce, for the difficulties that called for the comparatively rare and therefore richly rewarded commercial talent have disappeared. And someone must benefit from the reduction of the manufacturer's profit. Either goods must become cheaper, or, to put it the other way about, wages must rise. There is no other possibility." Gesell (1920), p. 135.

As pointed out above, the present paper complements research that has used modern economic theory to investigate whether the Gesell hypotheses can be replicated in standard model frameworks. One finds that the results of previous research are mixed.

For example, Rösl (2006) finds that only the first hypothesis can be derived from Sidrauski (1967), that is, in a money-in-the-utility set-up. He concludes that Gesell neglected an analysis of the long run and any possible effects on capital accumulation so the other three hypotheses turn out to be non-valid in his model.

In turn, Menner (2011), for example, uses an elaborate and involved New Monetarist DSGE model to find that "inflation and 'Gesell taxes' maximize steady-state capital stock, output, consumption, investment and welfare at moderate levels. In a recession scenario, a Gesell tax speeds up the recovery in a similar way as a large fiscal stimulus but avoids 'crowding out' of private consumption and investment." Thus, he finds support for the Gesell hypotheses at moderate levels in his business cycle model of the third-generation monetary search models.

The present paper uses an alternative micro-founded and simple general equilibrium model to analyze whether the depreciation of money is socially beneficial. Doing this we will abstract from fiscal policy, as Gesell did not consider the interaction of fiscal and monetary policy in detail. [4]

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[4] If one likes, the results here may also interpreted as holding relative to some given and constant fiscal policy operating in the background, and Ricardian Equivalence holds. Furthermore, another word of caution should be mentioned. I will not address the historical and the more recent empirical experiences that, mostly, local experiments using money depreciation have produced. Of course, the most famous one is the Wörgel experiment from 1932 to 1933 which was stopped by the Austrian National Bank in

We follow Gesell and assume that there is homogeneous money that is issued by the state and by law that money is legal tender in transactions. [5] Furthermore, his advocated monetary system is one where fiat (paper) money is irredeemable, and, thus, directly related to (only redeemable by) real goods in the economy.

Irredeemability implies that you cannot exchange a banknote back into another banknote or any collateral that might back the face or any other (real) value of the banknote. For instance, in the Euro and the Fed system you can in principle redeem your banknote, but only to get another banknote with an equally denoted face value. This is not possible under the irredeemability of the banknote and plays a role when there is a depreciation of the face or other value of the banknote. On the issue of irredeemability and fiat money see, for example, Buiter (2003).

In the paper, the basic Sidrauski framework is changed importantly. Apart from the motive to derive utility from money it is assumed that agents also derive utility from their wealth. [6] People are taken to be rational and are not fooled by money illusion. Thus, the agents only consider real, physical capital as wealth.

Here I relate to this more general concept as 'love of wealth' in a dynamic macroeconomic model as in Rehme (2011). These motives are important for deriving nondegenerate short-run relationships between the nominal interest rate and micro-founded consumption (quasi-) IS and LM curves (in a "nominal interest rate and consumption" space). A similar approach based on the 'love of wealth' as a micro-foundation in a dynamic model with a short-run and long-run analysis as in this paper has recently been presented by Michaillat and Saez (2014). In this framework, I analyze two approaches to capture Gesell's ideas, which are presented in two parts of the analysis.

The first approach, the present Part I, concentrates on textbook-like short-run, demand-determined equilibria of the (quasi-, micro-founded) IS-LM-AS-AD variety, which are based on the micro-foundations of optimal behaviour, that is, the demand of the agents. The link to supply is assumed to be Keynes's "principle of effective demand".

September 1933. The interested reader will find a plethora of empirical evidence on whether money depreciation and Gesell's ideas in general work or not in the literature. Here the focus is on theory.

[5] Thus, the assumption implies that we rule out currency substitution. For an analysis of depreciating money in a complementary currency system see, for example, Godschallk (2012).

[6] This has been done, for example, by Weber (1930) and Pigou (1941) who argue that individuals derive utility from the mere possession of wealth and not simply its expenditure. Later Kurz (1968) provided a thorough analysis of an optimal growth model where wealth features in utility. Furthermore, Zou (1994), Bakshi and Chen (1996) and Carroll (2000) relate to Max Weber and argue that the dependence of utility on wealth captures the "spirit of capitalism". More generally, it captures 'love of wealth' as argued in Rehme (2017).

The second one, analyzed in Part II, complements the first approach and uses a standard Ramsey-Cass-Koopmans framework for the long run where markets are as-summed to clear at each point in time, and demand always equals supply.

The following results are then obtained for the first approach and so for Part I of the analysis. In a short-run, demand-determined equilibrium where the (physical) capital stock, the inflation rate, transfers, and money supply are fixed, but real factor prices are flexible, Gesell's hypotheses GC1 - GC4 are all valid, given the (demand) micro-foundations of the model and given that the micro-foundations feature direct utility derived from money transactions and 'love of wealth', where only physical capital is considered to be the true source of wealth.

A key assumption for the derivation of this result is that the marginal productivity theory of distribution does not necessarily hold in the short run. Importantly, when the inflation rate is given, and the Fisher relationship holds, the real interest rate moves in the same direction as the nominal interest rate in any short-run equilibrium. [7]

The details for this are presented in the main text. Thus, the real interest rate is determined by other factors than technology in the short run.

Gesell's ideas have been important in recent discussions about overcoming the zero lower bound that has played such an important role after the Great Recession. One argument has been to make nominal interest rates negative to combat what is called a "liquidity trap". For good surveys on this, its relation to Gesell's ideas, their relevance for the current economic situation and their historic precursors see, for example, Darity (1995), Ilgmann and Menner (2011) and Svensson and Westermark (2016).

In the present paper, it turns out that many different combinations of money depreciation and money supply policies can sustain a "liquidity trap", that is, a situation with a short-run equilibrium, zero nominal interest rate. These monetary policy combinations are shown to have non-negligible effects for distribution, that is, the rewards to labour and capital.

In various model variants, Buiter and Panigirtzoglou (2003) and other contributions by W. Buiter have shown that money depreciation may be used to make the short-run equilibrium interest rate negative and pull an economy out of a "liquidity trap". In this paper, I find the same so complementing their results. But the model structure here is quite different and simple. Given the present model's micro-foundations, this result follows straightforwardly and easily.

Another application of the model for the short run is the recent episode of demonetization in India where the 500 and 1000 rupee notes (INR) were declared invalid in a surprise move by

[7] Notice that the "Fisher relationship" captures that the nominal interest rate is (approximately) the sum of the real interest rate and (expected) inflation. This should not be equated with the "Fisher effect" which states that the real interest rate is independent of the rate of inflation. For this clarification see, for example, Ahmed and Rogers (1996). For textbook models where the real and the nominal interest rate move in the same direction in the short run, see, for example, Blanchard (2017), ch. 6 and 16.

the Indian Prime Minister. In India, cash is by far the most important medium for economic transactions. Overnight this affected 86.9 percent of the value of total currency in circulation.

The model predicts that such a demonetization leads to lower consumption, aggregate demand, and lower wages, but higher real interest rates. Thus, the measure does not seem to be good for workers in the short run. These findings are broadly in line with the empirical evidence documented by the Reserve Bank of India's Monetary Policy Department (MPD) (2016).

Summarizing these findings for the short run yields that the present model framework is indeed capable of verifying Gesell's claims. In the short-run, demand-determined equilibrium all claims can be ascertained. This may provide a rationale for why the renewed interest in his ideas plays a role in the current economic policy deliberations.

The paper is organized as follows. Section 2 presents the model and 3 analyzes the demand-determined (short-run) equilibrium, an overcoming of the zero lower bound on nominal interest rates and, as an example, applies the model to the recent Indian demonetization episode. Section 4 concludes.

2 The Model

To simplify the algebra the model is set in continuous time. For all variables that are continuous functions of time, I use the subscript t to denote their dependence on time. Thus, we define $h_t \equiv h(t)$ for some variable h depending on time. Furthermore, the change of a variable h over time, i.e. $\frac{dh_t}{dt}$, is denoted by $\dot{h}_t$.

By assumption, the economy is populated by many, price-taking households. The aggregate resource constraint of the households is given by

$$C_t + \dot{K}_t + \frac{\dot{M}_t}{P_t} + \sigma \cdot \frac{M_t}{P_t} = w_t N_t + r_t K_t + X_t \tag{1}$$

where C_t and K_t denote aggregate real consumption and the aggregate real capital stock, respectively. M_t represents the aggregate nominal money holdings and P_t is the price level. N_t denotes population and w_t is the real wage rate. r_t denotes the real rate of return on capital, net of depreciation of physical capital K_t. The lump-sum (real) transfers of the government that are granted to the households are denoted by X_t.

Thus, the right-hand side of the budget constraint (1) captures aggregate income, consisting of total wage $(w_t N_t)$ and capital income $(r_t K_t)$ as well as government transfers (X_t).

The left-hand side, in turn, captures aggregate spending. Income is spent on consumption (C_t), investment in new capital $(\dot{K}_t)$ and acquisitions of new, real money holdings $\left(\frac{\dot{M}_t}{P_t}\right)$.

The aggregate budget constraint in equation (1) corresponds to the conventional set-up of a Sidrauski (1967), money-in-the-utility-function model. The novel feature and, for this paper, the crucial difference is the term $\sigma \cdot \frac{M_t}{P_t}$. It captures the *Gesell tax*, that is, the idea of "*rusting money*". That can be interpreted as a depreciation on the circulating real money holdings of the households and is tantamount to a tax on them.

Sometimes it is argued that the Gesell tax is simply another form of an inflation rate that most people also consider a tax on money holdings. But notice that the Gesell tax is directly determined by a political entity such as e.g. a central bank, and not, like the inflation rate (tax), indirectly by the workings of markets.

Now consider a representative agent economy, and define per capita consumption c_t, real money balances m_t, as well as the per capita capital stock k_t and transfers x_t as follows

$$c_t \equiv \frac{C_t}{N_t},\ m_t \equiv \frac{M_t}{P_t N_t},\ k_t \equiv \frac{K_t}{N_t},\ \text{ and } x_t \equiv \frac{X_t}{N_t}$$

Dividing equation (1) by N_t and using our definitions then yields

$$c_t + \frac{\dot{K}_t}{N_t} + \frac{\dot{M}_t}{P_t N_t} + \sigma m_t = w_t + r_t k_t + x_t$$

It is not difficult to verify that $\frac{\dot{K}_t}{N_t} = \dot{k}_t + n_t k_t$ and $\frac{\dot{M}_t}{P_t N_t} = \dot{m}_t + \pi_t m_t + n_t m_t$ where $\pi_t \equiv \frac{\dot{P}_t}{P_t}$ represents the rate of inflation and $n_t = \frac{\dot{N}_t}{N_t}$ the population growth rate. Then the budget constraint of the representative household is given by

$$c_t + \dot{k}_t + n_t k_t + \dot{m}_t + \pi_t m_t + n_t m_t + \sigma m_t = w_t + r_t k_t + x_t.$$

Again, the right-hand side corresponds to the household's income and the left-hand side captures the household's expenditure. Notice that σm_t can be regarded as an outlay for the household. The longer the household holds real money balances m_t, the more is foregone (a form of expenditure) in terms of real income. For a similar set-up see, for example, Rösl (2006). It captures what is called the Gesell tax.

Building on, for example, Blanchard and Fischer (1989), ch. 4.5, and the Rösl setup we now denote real per capita resources by a_t where $a_t \equiv k_t + m_t$. Thus, the household has real resources in the form of physical capital and real money balances. It follows that $\dot{a}_t = \dot{k}_t + \dot{m}_t$. After collecting terms and rearrangement Appendix B shows that one then obtains

$$\dot{a}_t = [(r_t - n_t)a_t + w_t + x_t] - [c_t + (r_t + \pi_t + \sigma)m_t] \tag{2}$$

Thus, the change in real per capita resources $\dot{a}_t$ depends on the household's income from capital and real money balances $(r_t - n_t)a_t$, labour income w_t and transfers x_t. Consumption then consists of the consumption of goods c_t and the expenses for using money services. The latter depends on the user cost of money $(r_t + \pi_t + \sigma)m_t$. Here we employ the Fisher relation that nominal interest rates i_t equal the real interest rate r_t plus the inflation rate π_t. The user cost of holding money, thus, depends on the nominal interest rate i_t and the depreciation of money σ.

To simplify the algebra consider an economy with no population growth $n_t = 0$ and a population set to $N_t = 1$ for all t. One easily verifies that the paper's qualitative results do not depend on these assumptions. Furthermore, notice that by assumption r_t represents the rate of return of physical capital net of depreciation. This will become clearer when presenting the firms' problem. [8]

As an important departing point from a standard Sidrauski model, the representative household is now taken to "love wealth". By assumption, the household is forward-looking and not fooled by money illusion. Thus, only physical capital is considered to be "wealth" that directly bears on welfare. Hence, k_t features in the utility function as, for example, in Kurz (1968). [9]

However, the household also values the fact that real money balances facilitate exchange and transactions. Thus, (real) money balances are also taken to bear on welfare as in Sidrauski (1967). Although both (fiat) money and capital feature directly in utility, they do so for different reasons. Money is valued because it facilitates exchange, whereas physical capital is valued as an expression of wealth. [10]

The household's problem is then taken to be to maximize the functional

$$W = \int_0^\infty \varphi(c_t, m_t, k_t) e^{-\rho t} dt \quad , \tag{3}$$

where $\varphi(c_t, m_t, k_t)$ is period utility depending on consumption, real money balances and physical capital. Welfare is discounted at the (positive) rate of time preference ρ, capturing how patient households are, and the convergence of the utility function.

One needs to put more structure on these preferences because Kurz (1968) has analyzed a neoclassical growth model with physical capital in the utility function (i.e. preferences with "love of wealth") and shown that the dynamic properties of such a model are extremely

[8] Otherwise, some slight adjustments of the model after reintroduction of a positive n would also serve the purpose of working with a net return on capital, because n can also be interpreted as a factor that corresponds to some form of social depreciation rate in a simple Solow model. This argument can be found in almost any elementary textbook on macroeconomics.

[9] The expression love of wealth is based on Plutarch's (46 AD - 120 AD) essay " Περι ` φιλοπλοντιας " ("De Cupiditate Divitiarum" or "On the Love of Wealth") in his Moralia.

[10] For a clarification why money may be taken to feature directly in utility cf. Feenstra (1986).

cumbersome to analyze. Furthermore, no clear results appear to be obtainable if allowing for the more general setups. [11]

To derive clear predictions that also allow for an analysis of transitional dynamics, and building on previous own work, cf. Rehme (2011), we now make the following assumptions about the period utility function $\varphi(c_t, m_t, k_t)$.

1. $\varphi(c_t, m_t, k_t)$ is taken to be separable in c_t, m_t and k_t. In particular, assume that $\partial^2\varphi(\cdot)/\,\partial i\,\partial j = 0$ for all $i, j = c_t, m_t, k_t$ and $i \neq j$.
2. $\varphi(c_t, m_t, k_t)$ is increasing and concave in each (own) argument, that is, $\partial\varphi(\cdot)/\,\partial i > 0$ and $\partial^2\varphi(\cdot)/\,\partial i^2 < 0$ for all $i = c_t, m_t, k_t$. [12]
3. $\varphi(c_t, m_t, k_t)$ satisfies the Inada conditions for each (own) argument, that is,

$$\lim_{i\to 0}\varphi(\cdot)/\,\partial i \to \infty \text{ and } \lim_{i\to\infty}\varphi(\cdot)/\,\partial i \to 0 \text{ where } i = c_t, m_t, k_t$$

Thus, as in Sidrauski (1967), period utility depends on (per capita) consumption c and real money balances m. What is different is that the agent additionally derives welfare from (per capita) wealth (capital). [13] Thus, $\varphi(\cdot)$ is also a function of k. That captures that many people value wealth and capital per se. For instance, many people like to look at and visit impressive buildings, e.g. the Eiffel Tower, the Empire State Building or the like, and derive utility from that. [14]

The separability assumption is often invoked. It means that the decision on one of the variables does not depend on any of the other variables. Thus, the agent focuses only on one variable when making plans. That, of course, does not imply that the optimal choice is independent of the other variables because they are linked through the budget constraint. On separation approaches in economic modeling see, for example, Blackorby, Primont, and Russel (2008) or Acemoglu (2009), ch. 10.1.

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[11] In fact, the more general a setup, the more empty the content of a model may often be.

[12] A positive marginal utility of wealth ∂φ(·)/∂k_t>0 is necessary for a non-degenerate IS curve.

[13] The question arises whether it is relative wealth (status concerns) or absolute wealth that matters for individuals. The former played a great role for preferences according to, for example, Smith (1759).

[14] Also, many firms offer guided tours through their often very impressive plants of production such as e.g. the Boeing assembly halls in Seattle or Volkswagen's "Auto Manufaktur" in Dresden. Clearly, marvelling at buildings from outside means that these buildings or plants have a public good nature. However, visiting them usually requires a fee to be paid so that buildings and guided tours then have a private good nature.

Taking welfare to be increasing in wealth is perhaps more problematic. Clearly, there are cases where additional capital may be valued less. An example may be an additional nuclear power plant. However, k is an index of all sorts of capital stocks. Most evidence would suggest that people generally like wealth. Otherwise, they would not do the things one can observe to increase their wealth. Of course, this is a perennial phenomenon. Thus, drawing on this "stylized fact" may justify the assumption that welfare is increasing in wealth, $\varphi_k > 0$.

Assuming that the welfare gain becomes smaller as wealth increases captures the observation that very rich people often say that an additional "yacht" may not make them much happier, especially in comparison to the first one they already own.

The Inada conditions on welfare's reaction to the effects of wealth when there is hardly any or too much capital are not necessary for most of the analysis below but can be rationalized on quite intuitive grounds. For example, $\lim_{k\to 0}\varphi_k = \infty$ would say that one is craving wealth if one does not have any. In turn, $\lim_{k\to\infty}\varphi_k = 0$ would imply that Bill Gates does not really care if he gets an additional computer.

A simple and convenient period utility function that satisfies all these requirements is the logarithmic one. So we invoke

Assumption 1 *Period utility $\varphi(c_t, m_t, k_t)$ is separable and logarithmic in each argument and given by*

$$\varphi(c_t, m_t, k_t) = \ln\, c_t + \delta \ln\, m_t + \beta \ln\, k_t \quad \text{where} \quad \delta, \beta > 0 \tag{4}$$

The parameter δ measures how people value the transaction services real money balanced render, and β captures "love of wealth". The assumption that δ and β are positive means that the model is structurally different from the more conventional setups of "money-in-the-utility-function"-models without "love of wealth".

From the logarithmic utility set-up it is immediate that relative wealth, for instance, the logarithm of the ratio of individual to total (aggregate) wealth would be separable in the two concepts. If the representative individual takes total wealth as given, then both approaches, that is, working with relative or absolute wealth would not make a difference in the individual's decision and would yield similar results. As argued above I follow Plutarch here;

Let $[h_t]_{t=0}^{+\infty}$ denote the continuous time path of variable h_t and use the following definitions: $k_t \equiv (1 - z_t)a_t$ and $m_t \equiv z_t a_t$ where a_t is an indicator of the total real resources of the household, and z_t denotes the share of the real resources held in terms of real money balances. These definitions serve to facilitate the analysis, and i.a. imply

$$\begin{aligned} \varphi(c_t, m_t, k_t) &= \ln\, c_t + \delta \ln\,[z_t \cdot a_t] + \beta \ln\,[(1 - z_t) \cdot a_t] \\ &= \ln\, c_t + (\delta + \beta) \ln\, a_t + \delta \ln\, z_t + \beta \ln\,[(1 - z_t)] \end{aligned} \tag{5}$$

We can then formulate the representative household's problem as the maximization of intertemporal welfare based on (5) subject to the flow budget constraint in (2). Thus, the household's problem is

$$\max_{c_t, z_t} \int_0^\infty [\ln\ c_t + (\delta+\beta)\ln\ a_t + \delta\ln\ z_t + \beta\ln(1-z_t)]e^{-\rho t}dt$$
$$\text{s.t.}\quad \dot{a}_t = [r_t a_t + w_t + x_t] - [c_t + (r_t + \pi_t + \sigma)z_t a_t]$$

Here consumption c_t and real money balances m_t in terms of per capita resources a_t, that is, z_t are the control variables, and a_t is the state variable. The household takes the paths of the real interest rate, the wage rate, the inflation rate and government transfers $[r_t, w_t, \pi_t, x_t]_{t=0}^{+\infty}$ and the (constant) policy parameter σ as given. Recall that $n_t = 0, \forall t$, (no population growth) has been assumed. Furthermore, the household takes as given his initial level of real resources, a_0.

To solve the consumer's problem, we set up the current-value Hamiltonian

$$\begin{aligned}\mathcal{H} =& \{\ln\ c_t + (\delta+\beta)\ln\ a_t + \delta\ln\ z_t + \beta\ln(1-z_t)\}\\ &+\mu_t[r_t a_t + w_t + x_t - c_t - (r_t+\pi_t+\sigma)z_t a_t)]\end{aligned}$$

where μ_t is the current-value costate variable. [15]The necessary first order conditions for this maximization problem are

$$\frac{1}{c_t} - \mu_t = 0 \tag{6}$$

$$\frac{\delta}{z_t} - \frac{\beta}{1-z_t} - \mu_t \cdot a_t(r_t + \pi_t + \sigma) = 0 \tag{7}$$

$$-\left[\frac{\delta+\beta}{a_t} + r_t\mu_t - \mu_t(r_t+\pi_t+\sigma)\cdot z_t\right] = -\rho\mu_t + \dot{\mu}_t \tag{8}$$

where we also require that equation (2) holds (with $n_t = 0$) and the transversality condition is satisfied, i.e.

$$\lim_{t\to\infty} \mu_t \cdot a_t \cdot e^{-\rho t} = 0 \tag{9}$$

Recalling the definition of z_t with $m_t \equiv z_t \cdot a_t$ and $k_t \equiv (1-z_t)\cdot a_t$ and using equation (6) one can simplify equation (7) to

[15] For what is to follow we now use subscripts, except subscript t, to denote partial derivatives.

$$\frac{\delta}{z_t a_t} = \frac{\beta}{(1 - z_t) \cdot a_t} + \mu_t \cdot (r_t + \pi_t + \sigma)$$

$$\frac{\delta c_t}{m_t} = \frac{\beta c_t}{k_t} + (r_t + \pi_t + \sigma) \tag{10}$$

which implicitly describes the demand for real money balances m as is shown below. [16]
The equations (6) and (8) with $z_t = m_t/a_t$ imply that

$$\frac{\dot{c}_t}{c_t} = \frac{(\delta+\beta)c_t}{a_t} + r_t - \frac{(r_t+\pi_t+\sigma)m_t}{a_t} - \rho \tag{11}$$

whereby consumption growth depends on "love of wealth" and the preference for money holdings. Unlike in conventional models the stocks of money and physical capital, which feature in a_t, bear on the growth rate of consumption. Notice that in this model there is, in general, a wedge between the real interest rate r_t and the time preference rate ρ. It is not difficult to see that in a steady state when $\dot{c}_t = 0$ the real interest rate r_t will in general not be equal to the time preference rate.

Using equation (10) where

$$\frac{\delta c_t}{m_t} - \frac{\beta c_t}{k_t} = (r_t + \pi_t + \sigma)$$

the expression for the consumption growth rate in equation (11) boils down to

$$\begin{aligned}\frac{\dot{c}_t}{c_t} &= \frac{(\delta + \beta)c_t}{a_t} + r_t - \left(\frac{\delta c_t}{m_t} - \frac{\beta c_t}{k_t}\right)\frac{m_t}{a_t} - \rho \\ &= \frac{\beta c_t}{a_t} + \left(\frac{\beta c_t}{k_t}\right)\frac{m_t}{a_t} + r_t - \rho = \frac{\beta c_t}{a_t}\left[\frac{k_t + m_t}{k_t}\right] + r_t - \rho\end{aligned}$$

Thus, the growth rate of consumption is given by

$$\frac{\dot{c}_t}{c_t} = \beta\left(\frac{c_t}{k_t}\right) + r_t - \rho \tag{12}$$

which shows that "love of wealth", i.e. β is an important determinant of the consumption growth rate. In particular, if β is zero, we are back to the conventional and simplest money-in-the-utility model, where the economy dichotomizes into a real and nominal sector. This is

[16] Notice that for a conventionally shaped LM curve below one has to invoke the (mild) assumption that $\delta/\beta > m/k$. See also section 3.

because, if that is the case, in steady state $r_t = \rho$. But here with a β that is taken to be non-zero, consumption growth depends on how people value capital.

3 A demand-determined (short-run) equilibrium

In this section, we take the representative household's optimum to describe the micro-foundations of aggregate demand. These foundations are, of course, based on the preferences postulated in assumption 1. Thus, suppose the short run is described by the demand side of the economy so that Keynes's "Principle of Effective Demand" would hold. To fix ideas assume that the capital stock, the real money supply and prices (inflation rate) are fixed in the short run, possibly at their steady-state levels. We can then conduct a simple thought experiment which is similar to a conventional, textbook-like IS-LM analysis. To do this, we now drop time subscripts for variables in steady state and proceed as follows.

Equation (10) describes the choice of z_t and so implicitly the choice of (irredeemable) real money balances m, and yields the model's demand for money when assuming that the Fisher relation $i = r + \pi$ holds.

$$m^d = \frac{\delta \cdot c}{i + \sigma + \beta \cdot \frac{c}{k}} \tag{13}$$

Notice that money demand here depends negatively on the nominal interest rate, but the latter can be zero and money would still be demanded, when β and σ are nonzero. Furthermore, it is not difficult to verify that money demand depends positively on consumption c, which reflects the transaction motive of money demand.

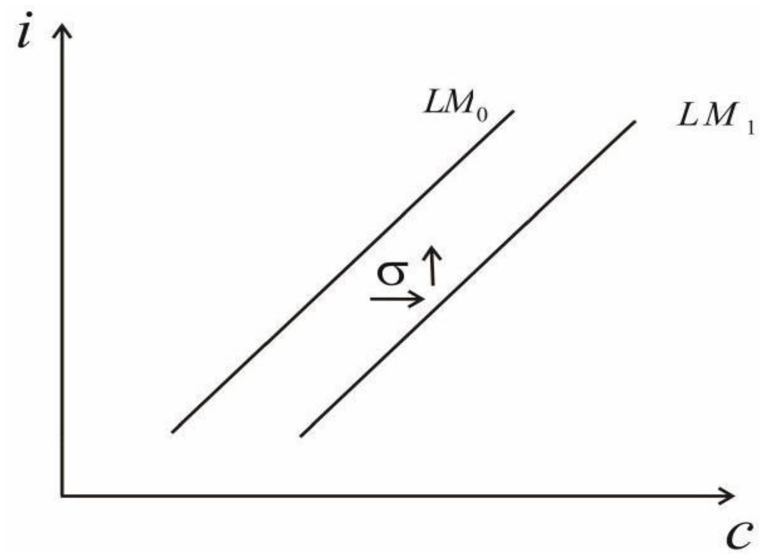
We follow Gesell as closely as possible below and assume that the money market is in equilibrium, money is irredeemable and the only legal tender in the economy. To this end money supply, m^s, is taken to be exogenously determined by the monetary authority, and, importantly, taken to equal money demand m^d. For simplicity use m to convey this from now on. Thus, $m = m^s = m^d$ is assumed.

But from equation (10) one then also obtains a quasi LM-curve in consumption c and the nominal interest rate i. To get a rather conventionally shaped relationship between c and i assume that $\delta/\beta > m/k$, then the quasi-LM curve is given by;

$$LM\colon\ c = (i + \sigma)\left[\frac{\delta}{m} - \frac{\beta}{k}\right]^{-1}. \tag{14}$$

Similarly to a textbook LM curve, one gets $dc/di_{|LM} > 0$ and $dc/dm_{|LM} > 0$. Thus, in a (c, i)-space the LM has a positive slope in terms of the nominal interest rate i and is shifted to the right when the money supply increases. Importantly for this paper, the LM is also shifted to the right in a (c, i)-space if σ is increased, that is, $dc/d\sigma_{|LM}$ for a given nominal interest rate.

Figure 1: The LM Curve



Result 1 (LM Curve): Based on the household's optimality conditions, equation (14) describes an LM curve in (c, i)-space for a given capital stock k, and fixed money supply m and inflation rate π. It expresses consumption as a function of the nominal interest rate $i = r + \pi$, depending on real money balances m and the money depreciation rate σ. It describes equilibrium in the money market. An increase in the Gesell tax σ or real money balances m shifts the LM curve to the right in a (c, i)-space, for a given nominal interest rate

Next, we consider equation (11) which, as one should recall, is entirely based on the demand side of the economy, i.e. the households' optimality conditions. In steady-state that equation reduces to

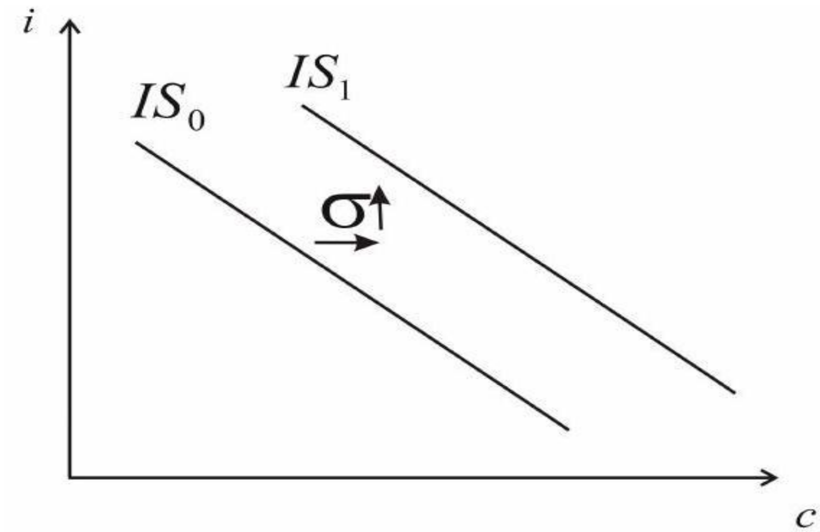
$$(\delta + \beta)c = (r + \pi + \sigma)m - ra + \rho a$$

After some manipulation, using the Fisher relationship $i = \pi + r$, can be rearranged to yield;

$$IS\colon\ c = \left(\frac{1}{\delta+\beta}\right)[(\pi + \sigma)(m + k) - (i + \sigma)k + \rho(m + k)] \qquad (15)$$

This equation amounts to a quasi-IS curve that has a negative slope with respect to i in a $(c, i) -$ plane. Thus, $dc/di_{|IS} < 0$. Furthermore, as wealth considerations play a role, i.e. $\beta > 0$, it turns out that the IS schedule also depends on real money balances. That is so because through the introduction of preferences for wealth (physical capital) the model also implies a Pigou effect whereby money positively bears on (real) consumption, i.e., $dc/dm_{|IS} > 0$. [17]

Figure 2: The IS Curve



The same holds for an increase in the inflation rate π, that is, $dc/d\pi_{|IS} > 0$. One also verifies that $dc/d\sigma_{|IS} > 0$. Thus, apart from an increase in real money balances m, an increase in the Gesell tax (an increase in σ) also shifts the IS curve to the right for a given nominal interest rate.

Result 2 (IS Curve): *Based on the household's optimality conditions, equation (15) describes an IS curve in (c, i)-space for a given capital stock k, and fixed money supply and inflation rate. It expresses consumption as a function of the nominal interest rate $i = r + \pi$ and depends on real money balances m and the money depreciation rate σ. It describes equilibrium in the goods market. The IS curve features a Pigou effect. An increase in real money balances or the inflation rate raises consumption and shifts the IS curve to the right for a given i. An increase in the Gesell Tax shifts the IS curve to the right in a (c, i)-plane for a given nominal interest rate.*

[17] Pigou (1943) argues that output and employment can be stimulated by increasing consumption due to a rise in real money balances. Later Patinkin (1948) coined the term for this effect after Arthur Cecil Pigou, one of the teachers of John Maynard Keynes.

3.1 The short-run, demand-determined equilibrium

As is well known from elementary macroeconomics, the intersection of the LM and IS curves describes a short-run, demand-determined equilibrium. From now on let a variable $h = h(t)$ in short-run equilibrium be denoted by $\hat{h}$.

Then solving equation (14) for the nominal interest rate i plus σ, inserting the result into the IS equation (15) and rearrangement yields the aggregate (short-run) demand for goods $\hat{c}$. In appendix D it is shown to be given by

$$\hat{c} = \frac{(\pi+\sigma+\rho)\cdot m}{\delta} \tag{16}$$

Using equation (14) one verifies that the (short-run) equilibrium nominal interest rate satisfies

$$\hat{\imath} = (\pi + \sigma + \rho)\left[1 - \left(\frac{m}{k}\right)\left(\frac{\beta}{\delta}\right)\right] - \sigma \tag{17}$$

where the expression in the square bracket is non-negative by assumption.

One can calculate the velocity of money as the ratio of $\hat{c}$ to real money balances m. As both quantities are expressed relative to the price level, the velocity of money (in terms of consumption) in a short-run equilibrium is then given by

$$\hat{v} \equiv \frac{\hat{c}}{m} = \frac{(\pi+\sigma+\rho)}{\delta} \tag{18}$$

which is increasing in σ, and captures Gesell's idea that controlling the velocity of money has a direct bearing on aggregate (real) demand.

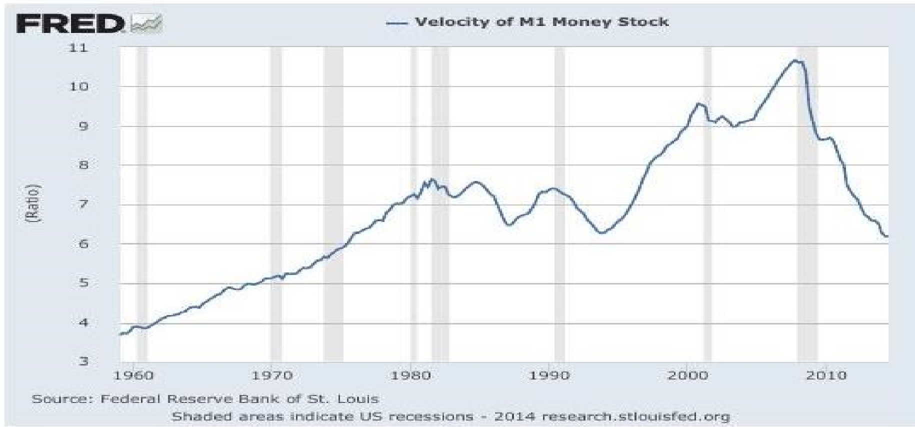
The velocity of money is usually larger than one which I assume to be the case.

Assumption 2: *The velocity of money, in terms of consumption, is taken to be larger than one, that is, $v > 1$ and, thus, δ to be sufficiently smaller than $\rho + \pi + \sigma$.*

Thus, the ratio of consumption - or more conventionally GDP - to money aggregates like M_0 (base money) or M_1 is taken to be a value over one. As an example consider the velocity of M_1 in the U.S. between 1960 and today. [18]

[18] *Money Velocity*: Velocity is a ratio of nominal GDP to a measure of the money supply (M1 or M2). It can be thought of as the rate of turnover in the money supply, that is, the number of times one dollar is used to purchase final goods and services included in GDP. Source: http://research.stlouisfed.org/fred2/categories/32242

Figure 3: Velocity of M_1 in the U.S.



Source: http://research.stlouisfed.org/fred2/categories/32242

From the graph, the velocity of M_1 has consistently been larger than one over the period considered. In this model, M refers to M_0 (base money). It is well known that the velocity of M_0 is usually higher than the one for M_1 because $M_0 < M_1$. As aggregate consumption corresponds to roughly 60 percent of GDP in most, especially OECD countries, it is safe to say that empirically the ratio of M_0 to aggregate consumption is also larger than one. This holds no matter whether we look at the steady state or shorter periods.

Given the expressions for a demand-determined equilibrium various comparative static investigations are then possible. As the paper's focus is on Gesell's conjectures, I concentrate on the effects on the short-run equilibrium if σ or m is changed. For now, assume that the inflation rate is non-negative, that is, $\pi \geq 0$.

From equations (16) and (17) aggregate demand for goods (in short run-equilibrium) is increased and the short-run equilibrium nominal interest rate falls when the Gesell tax (given real money balances) or real money balances (given money depreciation) rise. Thus, when the inflation rate is non-negative, we have;

$$d\hat{c}/d\sigma > 0,\ d\hat{\imath}/d\sigma < 0 \text{ and } d\hat{c}/dm > 0,\ d\hat{\imath}/dm < 0 \qquad (19)$$

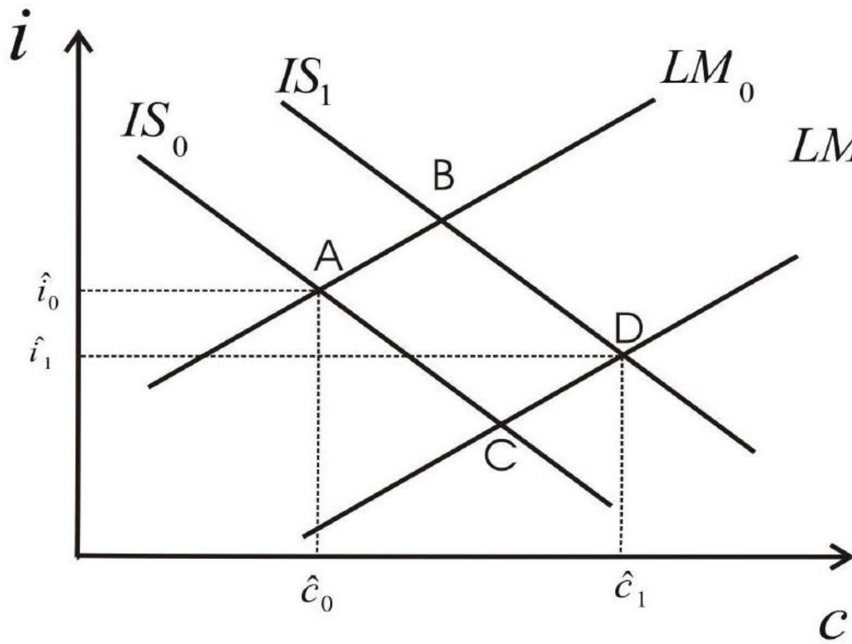
That means the (negative) nominal (short-run equilibrium) interest rate reaction to a positive change in the Gesell tax (σ) is larger in absolute value for the LM shift than the absolute (but positive) shift in the IS curve. This follows because $di/d\sigma_{|IS} = m/k$ and $di/d\sigma_{|LM} = -1$, and by the assumption that $m < k$.

Thus, if the economy's short-run equilibrium is initially at point A, an increase in σ or real money balances will move the LM and the IS curve to the right, to end up at a point like D with a higher $\hat{c}$ and a lower nominal interest rate $\hat{\imath}$ in the new short-run, demand-determined equilibrium.

Proposition 1: *Suppose the capital stock, prices, the inflation rate, and the transfers are fixed in the short run. Then an increase in the Gesell Tax σ, for a given nominal money supply,*

1. *increases the velocity of money $\hat{v}$, and*
2. *increases short-run, aggregate consumption $\hat{c}$, and*
3. *implies a lower short-run nominal interest rate $\hat{\imath}$ in a (quasi-) IS-LM environment in a (c, i)-plane.*

Figure 4: The Short-Run, Demand-Determined Equilibrium



By similar arguments, we also obtain that, for a given σ and $\pi \geq 0$, an increase in real money balances, m, increases short-run, aggregate consumption, $\hat{c}$, and implies a lower short-run nominal interest rate, $\hat{\imath}$.

So far we have ignored that the household's budget constraint, that is, equation (2) must also be satisfied. We consequently need that

$$r \cdot k + w + x - c - (\pi + \sigma) \cdot m = 0.$$

For convenience denote variables that are fixed in the short run by an upper bar. [19]

Assume that in a demand-determined (short-run) equilibrium the sum of wages and capital income equals output, called $\hat{y}$, which equals aggregate supply. Then $\hat{y} = r \cdot \bar{k} + w$. Given the determination of consumption by the IS-LM apparatus and in light of the budget constraint equation (2) we get

$$\hat{c} + (\bar{\pi} + \sigma) \cdot \bar{m} - \bar{x} = \hat{y}(r, w, \bar{k}) = r \cdot \bar{k} + w \tag{20}$$

where the left-hand side denotes aggregate demand (net of fixed and given transfers $\bar{x}$) and the right-hand side is a quasi-aggregate supply relationship that depends on the fixed capital stock $\bar{k}$, and the factor prices r and w.

If the factor prices are taken to vary freely and are not tied to marginal productivity remuneration, but some other exogenous process that is independent of k, it is indeed possible that the left-hand side of the equation, that is, aggregate demand, called ad, determines the right-hand side of the equation.

Letting $ad \equiv \hat{c} + (\bar{\pi} + \sigma) \cdot \bar{m} - \bar{x}$ denote aggregate demand, we have in a (short-run) demand-determined equilibrium that

$$ad(\sigma; \bar{m}, \bar{\pi}, \bar{x}) \equiv \hat{c} + (\bar{\pi} + \sigma) \cdot \bar{m} - \bar{x} = \hat{y}(r, w; \bar{k})$$

As a consequence, we can then define the following:

Definition 1 *Based on the household's optimality conditions in equations (10), (11), and (2), a short-run, demand-determined equilibrium is given when aggregate demand ad* $(\sigma; \bar{m}, \bar{\pi}, \bar{x})$ *equals aggregate output (supply),* $\hat{y}(r, w; \bar{k})$, *for a given capital stock, given real money balances and inflation rate. For flexible factor prices* r *and* w, *the intersection of IS and LM determines aggregate demand ad (...) and with it output* $\hat{y}(r, w; \bar{k})$ *so that the equilibrium is demand-determined.*

Whatever the values of the fixed variables and the parameters may be, the factor prices can equilibrate short-run demand and "supply" in such a world. Notice that we have not invoked the marginal productivity theory of distribution in which case the rewards would ultimately be functions of k. Instead, here we think of r and w determined by (e.g. market) forces outside the

[19] Recall that the IS-LM apparatus holds for a simultaneous equilibrium in the goods and money market. In that sense a given supply money makes it an exogenous variable for most of the analysis in this part of the paper.

model, but still assume that they equilibrate demand and supply in the way required by the model. If that is the case, ad $(\cdot)$ indeed determines "supply" $\hat{y}(r, w; \bar{k})$. [20]

Proposition 2*: Suppose the capital stock, output prices, the inflation rate, the transfers, and the money supply are fixed, but real factor prices are flexible in the short run. Then a short-run, demand-determined equilibrium, when the inflation rate is non-negative, is characterized by*

$$\text{ad}\,(\sigma; \bar{m}, \bar{\pi}, \bar{x}) = \hat{y}(r, w; \bar{k})$$

An increase in the Gesell Tax σ or real money balances then increases short-run, aggregate demand, ad, and consequently short-run output and supply, $\hat{y}$.

The properties easily follow from equation (20). From the proposition, we can also deduce the following. If σ rises, it follows from Proposition 1 that the (short-run) equilibrium nominal interest rate $\hat{\imath}$ falls. If the inflation rate is fixed in the short run, then the real interest rate r would have to fall. This follows from the Fisher relation $i = r + \pi$. If we assume that the factor prices are free to move in the short run, then Proposition 2 implies that the wage rate w must rise when σ increases. Thus, a higher σ implies a lower r, for a given $\bar{\pi}$, and higher ad so a higher $\hat{y}$ and a higher w. Hence, for a given capital stock, labour input and inflation rate, the wage earners would benefit from an increase in the Gesell tax.

Corollary 1: *For fixed capital, labour input and inflation rate, the wage earners may benefit from an increase in the Gesell tax or real money balances in the short-run, demand-determined equilibrium environment. The capital owners may earn less in such an environment.*

Of course, that begs the question of the factor prices are really more flexible than output prices, which determine the inflation rate π. Clearly, this distributional implication may not hold if the inflation rate is not fixed in the short run.

Lastly, the welfare implications in the short-run, demand-determined environment are considered. Clearly, if the money supply and capital stock are fixed in the short run, period (short-run) welfare from equation (4) is given by

[20] It is interesting to note that there may be many different combinations of w and r that can equilibrate ad and ŷ. Hence, under the assumptions made many different distributional arrangements for the rewards to capital and labour are feasible, and so the income distribution would in general not be determinate.

$$\hat{\varphi}(\hat{c},\bar{m},\bar{k}) = \ln\ \hat{c} + \delta\ln\ \bar{m} + \beta\ln\ \bar{k}$$

But then one easily verifies that $d\hat{\varphi}/d\sigma = (d\hat{c}/d\sigma)/\hat{c} > 0$, because $d\hat{c}/d\sigma > 0$. Thus, period welfare would rise with an increase in σ.

Proposition 3: *Suppose the capital stock, the inflation rate, transfers, and money supply are fixed, but real factor prices are flexible in the short run. Then a short-run, demand-determined equilibrium is characterized by period welfare*

$$\hat{\varphi}(\hat{c},\bar{m},\bar{k}) = \ln\ \hat{c} + \delta\ln\ \bar{m} + \beta\ln\ \bar{k} \ \text{ with } \ \frac{d\hat{\varphi}}{d\sigma} = \frac{d\hat{c}/d\sigma}{\hat{c}} > 0, \frac{d\hat{\varphi}}{d\bar{m}} = \frac{d\hat{c}/d\bar{m}}{\hat{c}} + \frac{\delta}{\bar{m}} > 0$$

that is, period welfare is higher, when the Gesell tax or real money balances are higher in a (short-run) demand-determined equilibrium.

The most interesting implication of the propositions for the short run is that Gesell's conjectures are true in the environment developed in this section. Thus,

Theorem 1: *In a short-run, demand-determined equilibrium where the capital stock, the inflation rate, transfers, and the money supply are fixed, but real factor prices are flexible and the inflation rate is non-negative, Gesell's hypotheses GC1 – GC4 are all generically valid, given the (demand) micro-foundations in equations (2), (4), (6), (7), (8), and (9), and given that the micro-foundations feature direct utility derived from money and "love of wealth" where physical capital is considered to be the true source of wealth.*

This result is striking and in contrast to some contributions in the literature. Clearly, the theorem is based on the non-implausible assumptions invoked here. Notice that the theorem is about the short run. However, Gesell's ideas have occupied the imagination of researchers and policymakers alike in the years right after the Great Recession. It has been and, somehow still, is being felt that money depreciation may be one way out of important crisis problems, in the short and the longer run.

3.2 Liquidity trap and the zero lower bound on nominal interest rates

Recently, it has been an important question what monetary policy can accomplish, if the nominal interest rate is at its zero lower bound, that is, if it takes on a value close to zero. As mentioned above there has been renewed interest in Gesell's ideas. To shed some led onto why

Gesell's ideas may be relevant in the current situation consider money demand and short-run equilibrium again. [21]

Money Demand Conditions

Consider a situation where the nominal interest rate is at the zero lower bound. Let us again concentrate on equation (7), which describes the optimal choice (demand) of money holdings in the private sector. For simplicity continue to use m_t to denote real money balances demanded and supplied. Then;

$$\frac{\delta}{z_t} - \frac{\beta}{1-z_t} - \mu_t \cdot a_t(r_t + \pi_t + \sigma) = 0 \qquad (7)$$

So far we have concentrated on an interior solution implying that the equation above is satisfied as an equality. Suppose that that is not the case. In particular, suppose that the nominal interest rate is at its zero lower bound with $i_t = r_t + \pi_t = 0$.

By implication the real interest rate r_t, the inflation rate π_t or both might in principle be negative. But in the short-run equilibrium, the inflation rate is (exogenously) given by assumption so we take the real interest rate r_t to adjust when $i_t = 0$. Thus, the real interest rate may be negative. There is, for example, evidence for the U.S. that negative real interest rates are far from unrealistic as is shown e.g. by Eichengreen (2015), Figure 1, which I represent here for convenience. [22]

Now, for the ensuing analysis recall that $\mu_t = 1/c_t$ and $a_t = k_t + m_t$ where in this section now $m_t = m_t^d$. We can then investigate various cases.

Case 1: Suppose $i = 0, \sigma = 0$ and $\beta = 0$. Then the left-hand *side* of equation (7) becomes $\frac{\delta}{z_t} > 0$ so that $z_t \to 1$ is optimal. Given that $m_t = z_t a_t = z_t(k_t + m_t)$ we need that $m_t \to \infty$ for $m_t/(k_t + m_t) \to 1$. Thus, people would demand an infinite amount of money balances and hoard cash.

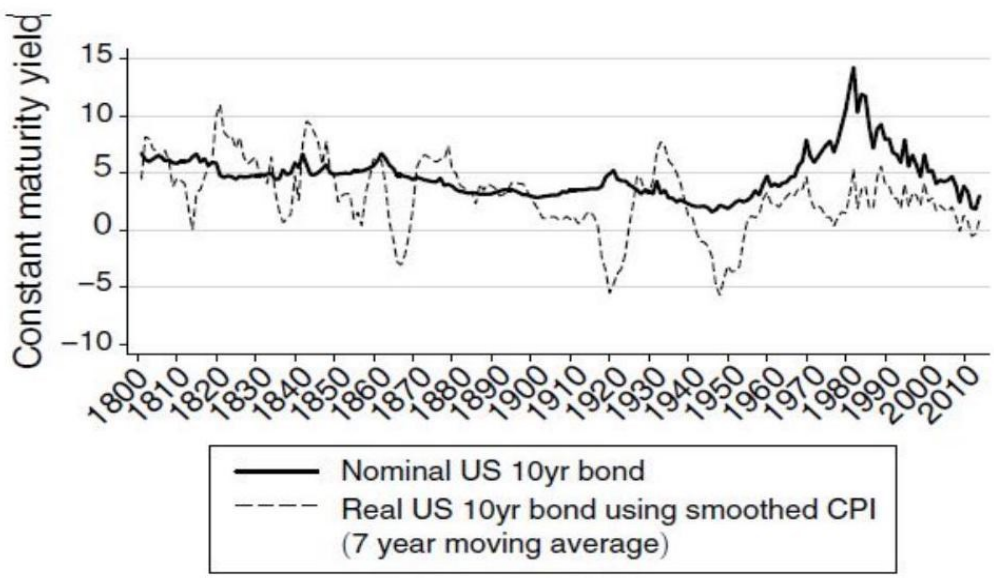
This is the conventional result following the Sidrauski model. The common explanation is that in a situation where the opportunity cost of holding money is nil, people would hold all

[21] The following analysis is also interesting for another reason. Gesell advocated a "free money" and "free land" economy. For those the interest rate would eventually have to abolished and any form of credit would be free of interest according to his utopia.

[22] More recent evidence for a range of countries can also be found in Desroches and Francis (20062007), Chart B1, and Yi and Zhang (2017), Figure 1.

their resources in the form of real money balances. That is usually associated with the notion of a “liquidity trap”. [23]

Figure 5: Long-Run US Interest Rates



Source: Eichengreen (2015), p. 66, Figure 1

Case 2: Suppose $i = 0, \sigma = 0$ and $\beta > 0$. Then equation (7) may yield an interior solution satisfying

$$\frac{\delta}{z_t} = \frac{\beta}{1 - z_t} \Leftrightarrow \frac{m_t}{m_t + k_t} = \frac{\delta}{\beta + \delta} \Leftrightarrow \frac{k_t}{m_t} = \frac{\beta}{\delta}$$

The important implication here is that $z_t < 1$ is optimal and so the presence of a “love of wealth”-motive (β) makes a liquidity trap less likely. That should be clear from the motive itself. If people value (physical) capital they will not try to get rid of all their capital in order to hoard only cash.

[23] The term and concept of a "liquidity trap" was well known by British economists before Keynes's publication of the "General Theory of Employment, Money and Interest", who actually never used the term himself. For details on that and some clarifications on misconceptions in current discourse on the phenomenon of a "liquidity trap" see Barens (2011).

In fact, pushing the argument further reveals that when the “love of wealth”-motive is extremely strong ($\beta \to \infty$) then people would want to get rid of all their money balances and only hold physical capital k_t [24] This seems to be not too unrealistic in view of the flight into real assets, that is, assets other than money which has been observed in many economies in the aftermaths of the Great Recession. [25]

Case 3: Suppose $i = 0, \sigma > 0, \beta > 0$. Then *equation* (7) must satisfy

$$\frac{\delta}{z_t} - \frac{\beta}{1 - z_t} - \left(\frac{k_t + m_t}{c_t}\right) \cdot \sigma = 0$$

which also implies an optimal z_t less than one.

When one takes the total differential of the left-hand side with respect to z_t and σ one obtains that $dz_t/d\sigma < 0$. Thus, a higher σ lowers the ratio $m_t/(k_t + m_t) = z_t$, and so either k_t is higher or m_t lower than the optimal z_t in case 2. Notice also that one gets a form of a (degenerate) LM-curve despite the fact that the nominal interest rate is at its lower bound, i.e. $i_t = 0$.[26]

The upshot of that is that the *Gesell tax* may stimulate investment in assets other than money, at least from the consumer’s perspective and when the nominal interest rate is at its zero lower bound. In that sense, the introduction of such a tax may stimulate investments in physical assets, especially if the “love of wealth” motive is not strong.

Proposition 4*: Suppose the short-run, nominal interest rate is at the zero lower bound $i_t = 0$. Then the household's optimality conditions for money balances demanded imply the following:*

When $\sigma = 0, i_t = 0$ and $\beta = 0$ the households all hoard cash when $\sigma = 0$ and $\beta = 0$, which corresponds to a "liquidity trap".

With "love of wealth" $(\beta > 0)$ an interior solution is possible where there is no hoarding of cash and households hold money and physical capital.

If the "love of wealth" is very strong (β is extremely large), people may move all their investments into physical capital and will get rid of all their money holdings.

The same effect may hold for a sufficiently large Gesell tax σ.

[24] This may be the case in an interior equilibrium with z_t=δ/(β+δ) or as a boundary solution δ/z_t -β/(1-z_t)< 0 of equation (7) when β→∞. Recently it has been argued that such a flight into real assets happened in the period where the nominal interest rate has indeed been at the zero lower bound. Thus, the model may provide a micro-founded explanation for this behaviour.

[25] It should be borne in mind, though, that this only holds if the money balances demanded actually satisfy the condition k_t/m_t=β/δ where by assumption k_t is fixed in the short run, that is, k_t=k ̅.

[26] Clearly, from equation (14) the LM curve is degenerate in this case, but there still is a money demand equation as can be gleaned from equation (13).

Implication 1

Notice that there are combinations for the monetary policy variables m and σ so that the short-run nominal interest rate is indeed zero in a short-run equilibrium. Then $i = \hat{\imath} = 0$ in equation (17) implies that

$$(\pi + \sigma + \rho)\left(\left(1 - \left(\frac{m}{k}\right)\left(\frac{\beta}{\delta}\right)\right) = \sigma \qquad (21)$$

must hold in the general case where β and σ are non-zero. That requires particular combinations for the monetary policy variables m^s and σ to sustain a zero, short-run equilibrium nominal interest rate. With that in mind, we now analyze the consequences for consumption and other real variables based on the cases considered above.

Case 1: If $\hat{\imath} = 0, \beta = 0$, and $\sigma = 0$, money *demand* is infinite, not well-defined and the LM-curve is a flat line. However, the model features a Pigou effect, the IS curve can, therefore, be shifted to the right, that is, consumption can be increased when the money supply m^s is increased. In that sense, monetary policy can be used to stimulate real demand and activity, even though $\beta = \sigma = 0$ and π is given. However, the demand for money will always be larger than the supply of it. Consequently, there is no equilibrium in the money market. One may argue that that puts pressure on prices and may cause inflation to rise. These results are not very surprising.

Case 2: If $\hat{\imath} = 0, \beta > 0$, and $\sigma = 0$, then by *equations* (7), (16) and (17) as well as concentrating on an interior solution, and under the assumption that the money supply m^s equals money demand m^d, we know that $m = \bar{k} \cdot (\delta/\beta)$ would have to hold. Based on that it is not difficult to verify that

$$\hat{c}_{|\sigma=0,\hat{\imath}=0} = \frac{(\bar{\pi} + \rho)\bar{k}}{\beta} = \frac{(\bar{\pi} + \rho)m}{\delta}$$

For a given capital stock and inflation rate, consumption can then not be stimulated by monetary policy. Furthermore, in an interior money market equilibrium the money balances, and especially the money supply, must satisfy $m = m^s = m^d = \bar{k} \cdot (\delta/\beta)$.

Case 3: If $\hat{\imath} = 0, \sigma > 0$, and $\beta > 0$, we *can* substitute for $(\pi + \sigma + \rho)$ from equation (21) in equation (16) to obtain

$$\hat{c}_{|\sigma>0,\hat{\imath}=0} = m \cdot \sigma \left[\delta - \frac{m}{k} \cdot \beta\right]^{-1}$$

Given that only certain combinations of m^s and σ sustain $\hat{\imath} = 0$, it is an interesting question whether these combinations have any real effects. It turns out that $dm/m = -[(\sigma/(\pi + \rho + \sigma)]d\sigma/\sigma$ must hold when $\hat{\imath} = 0$. See Appendix E. Thus, for example, a one-percent-increase in σ requires a corresponding $[(\sigma/(\pi + \rho + \sigma)]$ percent decrease in money supply m^s to keep $\hat{\imath}$ at the zero lower bounds.

In the appendix, it is then shown that the introduction of money depreciation coupled with a corresponding lower money stock when $\hat{\imath} = 0$ does not bear on consumption in equilibrium when the economy's interest rate is at the zero lower bounds.

However, aggregate demand may change. Recall that ad $(\sigma; m, \pi, x) = \hat{c} + (\bar{\pi} + \sigma)m_1 - \bar{x}$ when $\sigma > 0$ with $m_1 < m_0$, where m_0 denotes the money balances when $\sigma = 0$. A higher σ does not imply a higher $\hat{c}$, but it implies a larger $(\bar{\pi} + \sigma)m_1$ by the following arguments.

Taking logarithms one obtains $\ln(\bar{\pi} + \sigma) + \ln m_1$. The differential for this is $d\sigma/(\pi + \sigma) + dm_1/m_1$. For this expression to be positive it must be that

$$\left(\frac{\sigma}{\pi + \sigma}\right)\frac{d\sigma}{\sigma} + \frac{dm_1}{m_1} > 0$$

where $dm_1/m_1 = -(d\sigma)/\sigma \cdot \sigma/(\pi + \rho + \sigma)$. Making the substitution in the last inequality and rearranging implies a positive change in σ that

$$\left[\frac{\sigma}{\pi + \sigma} - \frac{\sigma}{\pi + \rho + \sigma}\right]\frac{d\sigma}{\sigma} > 0$$

must hold. Indeed it does because the expression in square brackets is positive. But then $d(ad)/d\sigma > 0$, which implies a higher $\hat{y}(r, w; \bar{k})$.

By the Fisher relation, we have $\hat{\imath} = \pi + r = 0$, that is, $\pi = -r$ when the nominal interest is at the zero lower bound for a given inflation rate. This ties down the (short-run equilibrium) real interest rate. Hence, an increase in σ implies an increase in the wage rate w. In that sense, the introduction of money depreciation has important distributional consequences when the nominal interest rate is at the zero lower bounds.

Proposition 5: *If the short-run, nominal interest rate is at the zero lower bounds, then a positive Gesell tax σ must be matched by a corresponding lower money supply to maintain $\hat{\imath}_t = 0$ and does not have effects on real equilibrium consumption but has positive effects on overall aggregate demand. In a short-run demand-determined equilibrium a higher σ with a corresponding lower m implies a relatively higher aggregate demand with a higher wage rate w and an unaltered real interest rate r.*

3.3 Overcoming the zero lower bound on nominal interest rates

It has recently been argued that a way out of the zero-lower-bound problem is to reduce the nominal interest rates to negative values. From the analysis up to now, that is a trivial consequence of the model.

Equation (19) tells us that an increase in m^s or σ would increase real consumption (increase the demand for goods) and lower the nominal interest rate, given that the inflation rate is non-negative. But the latter may not always be the case. In particular, in the aftermath of the Great Recession it was feared, and sometimes observed, that there was deflation, $\pi < 0$.

Thus, reconsider the equations (16), (17) and (19) which are given by

$$d\hat{c}/d\sigma = \frac{m}{\delta} > 0\,, \qquad \widehat{d\imath}/d\sigma = -\left(\frac{m}{k}\right)\left(\frac{\beta}{\delta}\right) < 0$$
$$d\hat{c}/dm = \frac{\pi+\rho+\sigma}{\delta}, \qquad \widehat{d\imath}/dm = -(\pi+\sigma+\rho)\left[\left(\frac{1}{k}\right)\left(\frac{\beta}{\delta}\right)\right].$$

One easily verifies that if deflation, $\pi < 0$, is strong so that $(\pi+\rho+\sigma) < 0$, then a change in m would produce an effect that may be unwanted, namely it would decrease consumption and raise the interest rate. This policy option may not be that attractive, especially if the monetary authority is not certain how strong deflation really is.

Thus, the other monetary policy instrument namely σ may be the more attractive to use, because an increase in money depreciation unambiguously raises consumption, and aggregate demand by equation (20) and lowers the nominal interest rate in a short-run equilibrium, irrespective of what the inflation rate is.

If the equilibrium interest rate $\hat{\imath}$ falls, then the real interest must fall too, when the inflation rate, be it positive or negative, is given. Thus, the equilibrium condition then implies that the wage rate must increase. Hence, again, this policy is good for the workers' income.

Proposition 6: *Suppose the short-run, nominal interest rate is at the zero lower bound $\hat{\imath} = 0$. Then, irrespective of the inflation rate π, an increase in the Gesell tax, σ, makes the short-run, nominal interest rate negative, $\hat{\imath} < 0$, increases equilibrium consumption, $\hat{c}$ and aggregate demand, ad. The real interest rate falls and the wage rate increases for equilibrium to hold.*

Thus, the model highlights arguments brought forward to combat the liquidity-trap situation that most economists agree has been around in recent years in, for example, the United States, Europe and Japan, in order to stimulate the real activity of the economy and raise welfare. Whether in reality, the effects on the factor income distribution are as predicted by the model here, cannot easily be ascertained. This is so because direct money depreciation was not used as a policy instrument.

Lastly one easily figures out the (similar) effects of changes in m on the shortrun equilibrium and the factor income distribution. However, it is still an unresolved empirical question whether labour has really benefitted more than capital when the economy is being pulled out of a "liquidity trap".

3.4 Demonetization

The model also allows for predictions on the (short run) effects of demonetization. For example, in the recent demonetization episode in India, 500 and 1000 rupee notes (INR) were declared invalid in a surprise move communicated on television on 8 November 2016 by Indian Prime Minister Narendra Modi. The Reserve Bank of India (RBI) set a period of fifty days until 30 December 2016 to deposit the demonetized banknotes as credit in bank accounts. [27]

The policy objective of the measure was to combat corruption, black and counterfeit money as well as terror financing. The demonetization affected 86.9 percent of the value of the total currency in circulation. Note that currency (cash) is by far the most important medium of monetary exchange in India. [28]

Demonetization can reasonably be identified with a reduction in the circulation of (base) money, that is, a decrease in m^s.

From the theoretical arguments above it follows what one should expect in this case. When, as in India for years before and after 2016, the inflation rate is positive, lower m^s implies reduced consumption (lower real demand for goods), $\hat{c}$, and generally a decrease in aggregate demand, $\widehat{ad}$, coupled with a higher nominal interest rate, $\hat{\imath}$, in a short-run equilibrium. Through the Fisher relation and for a given inflation rate, this means that the real interest rate $\hat{r}$ increases, but the real wage rate, $\widehat{w}$, decreases.

Proposition 7: *The short-run effects of a demonetization that decreases base money for a given inflation rate, lowers consumption and aggregate demand. It implies higher nominal and real interest rates, but a lower wage rate. It would benefit capital owners relatively more than workers.*

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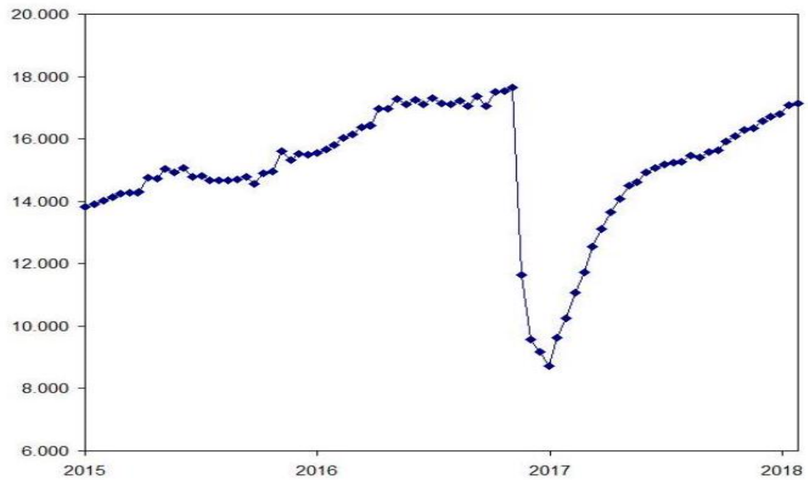
[27] The banknotes could also be exchanged at bank branches up to a limit that varied over the days. Initially, the limit was 4,000 INR per person from 8 to 13 November, then 4,500 INR per person from 14 to 17 November, and in the end 2,000 INR per person from 18 November. All exchange of banknotes was abruptly actually stopped from 25 November 2016 onwards.

[28] For example, some figures suggest that 99 percent of consumer transactions in India are carried out in cash and that the currency to GDP ratio is very high, but not the highest in international comparison. For instance, Japan's ratio is higher than India's. For that reason it is a natural experiment with empirical consequences that allows for thinking about the this issue using economic theory.

Figure 6: Demonetization in India 2016



Currency in Circulation | Billion Rupees | Weekly | last date: 2 Feb. 2018

Source: Reserve Bank of India Data Warehouse

The proposition captures very interesting aspects of (surprise) demonetization on an economy. But, of course, the model is too coarse to capture all ramifications of a policy measure that has had consequences on an economy as large as the Indian one.

A first, quasi-official assessment is provided by the Reserve Bank of India's (RBI's) Monetary Policy Department (MPD) (2016). According to that, there were important short-run negative effects after the policy announcement on some key sectors of the Indian economy, namely organized manufacturing (e.g. less vehicles, including three-wheelers sales, see Table 1), a drop in investment proposals, construction and other sectors. See p. 2-15 in that report.

As regards the effects on the nominal interest rate, the model may not do so well. Most evidence in the report shows that after the Indian demonetization move nominal interest rates (for most financial markets) fell. That would contradict the theoretical prediction. Of course, there are many indicators of "the" nominal interest rate in reality.

A drop in investment proposals may also be due to higher interest rates for business credits. Furthermore, stock market indices for India showed declines in sectoral performances, especially for the realty (property) sector after the policy move. See Table 16, p. 30. That might be an indication of lower nominal interest rates as well. However, the evidence for the other financial markets is not so clear.

But, as can be inferred from Figure 3, demonetization was quickly followed by remonetization, that is, by injections of new 500 and 2000 INR notes into the economy. As a consequence, most of the negative effects abided and the report of the Monetary Policy

Department (MPD) (2016) concludes that, all in all, the negative effects were "modest" over the short periodfrom November 2016 to February 2017.

Importantly, remonetization is running in the opposite direction of demonetization. Thus, we would expect the opposite of the short-run effects captured by Proposition 7.

Finally, it ought to be recognized that it is still an unresolved issue whether the policy objective to combat non-legal activities and transactions was successfully achieved by the Indian demonetization episode.

4 Conclusion

About one hundred years ago Silvio Gesell argued that money should 'rot' as any other good does. He advocated a monetary system, which he called "Free Money", where fiat (paper) money would be legal tender and irredeemable. He argued that depreciation of such money (cash) in circulation would be stimulative for economic performance and be socially beneficial.

In this paper, I question the claim that his ideas for an unconventional monetary policy cannot really be verified in modern economic theory frameworks. To this end I focus on four hypotheses Gesell made and analyze these using standard contemporaneous macroeconomic theory. The following findings of Part I of the analysis are then noteworthy.

In a short-run, IS-LM-AS-AD-like demand-determined equilibrium where the (physical) capital stock, the inflation rate, transfers, and money supply are fixed, but real factor prices are flexible, Gesell's hypotheses are all valid, given the (demand) micro-foundations of the model which feature utility directly derived from money and 'love of wealth', and physical capital is taken to be the true source of wealth.

This short-run analysis also implies that money depreciation can be a policy option to overcome the zero lower bound problems of nominal interest rates. Furthermore, an interpretation of the economic effects of the recent demonetization episode in India is possible from the model.

Hence, in the present model framework, Gesell's claims can be verified for a short-run environment. This may explain why there has been renewed interest in his way of thinking in the present economic, post-Great-Recession situation. One major insight of the analysis of Part I is, therefore, that in the short-run, demand-determined equilibrium all of Gesell's claims can be ascertained.

Of course, the analysis faces several caveats. The setup of the model is simple. Alternative utility and production functions might imply more complicated equilibria or the lack thereof. The introduction of fiscal policy may make the results less clean. 'Love of wealth' was captured by a constant. This begs the question of how changes over time in the 'love of wealth' may bear on the optimal paths. These and other extensions of the model are left for further research.

Appendix

A Absolute and relative wealth

Suppose the relative wealth of an individual i is given by;

$$x_i = \frac{k_i}{\sum k_j}$$

The relative wealth is the (absolute) level of wealth k_i in relation (relative) to total wealth (wealth of all people). If that individual's preferences are $u^i(c_i, x_i)$, then consumption c_i and relative wealth x_i would matter for person i 's welfare.

If there are many people, $j = 1, \ldots N$ where $i \in [1, N]$ with N very large, the effect of changes of k_i by individual i has no discernible bearing on total wealth $\sum k_j$ where the summation is from 1 to N. [29]

If the utility function of individual i is logarithmic,

$$u^i = \ln\ c_i + \gamma \ln\ x_i = \ln\ c_i + \gamma \ln\ k_i - \gamma \ln \sum\ k_j$$

then the decisions of individual i about c_i and k_i would not have an effect on $\gamma \ln\ \sum k_j$ which would be a datum for individual i. That follows from the assumption that there are many people. Those arguments justify what is mentioned in the text.

B Derivation of equation (2)

The steps leading to this equation are

$$\begin{aligned} c_t + (\dot{k}_t + \dot{m}_t) + (n_t k_k + n_t m_t) + \pi_t m_t + \sigma m_t &= w_t + r_t k_t + x_t \\ c_t + \dot{a}_t + a_t n_t + \pi_t m_t + \sigma m_t &= w_t + r_t k_t + r_t m_t - r_t m_t + x_t \\ c_t + \dot{a}_t + a_t n_t + \pi_t m_t + \sigma m_t &= w_t + r_t a_t - r_t m_t + x_t \end{aligned}$$

and so

$$\dot{a}_t = w_t + r_t a_t + x_t - a_t n_t - (r_t + \pi_t + \sigma) m_t - c_t.$$

Rearrangement yields equation (2) .

[29] This is almost always assumed in this literature. See, for example, Corneo and Jeanne (2001b).

C Derivation of equation (15)

From $(\delta+\beta)c = (r+\pi+\sigma)m - ra + \rho a$ one gets that

$$\begin{aligned}(\delta+\beta)c &= (r-r)m + (\pi+\sigma)m - rk + \rho(m+k)\\ &= (\pi+\sigma)m - rk + (\pi+\sigma)k - (\pi+\sigma)k + \rho(m+k)\\ &= (\pi+\sigma)(m+k) - (r+\pi+\sigma)k + \rho(m+k)\end{aligned}$$

which becomes equation (15) by the Fisher relationship $i = \pi + r$.

D Derivation of equation (16)

From equation (14) we get $c[\delta/m - \beta/k] = i + \sigma$. Substituting this in equation (15) implies

$$\begin{aligned}c(\delta+\beta) &= [(\pi+\sigma)(m+k) - c[\delta/m - \beta/k]k + \rho(m+k)]\\ &= (\rho+\pi+\sigma)(m+k) - c[\delta(k/m) - \beta]\\ c[(\delta+\beta) + \delta(k/m) - \beta] &= (\rho+\pi+\sigma)(m+k)\\ c[\delta + \delta(k/m)] &= (\rho+\pi+\sigma)(m+k)\\ c\delta\left[\frac{m+k}{m}\right] &= (\rho+\pi+\sigma)(m+k)\end{aligned}$$

Rearrangement then yields equation (16), that is, the expression for $\hat{c}$.

To obtain the expression for $\hat{\imath}$ substitute the last expression for c_t in equation (14) to get;

$$\begin{aligned}\frac{(\pi+\sigma+\rho)\cdot m}{\delta} &= (i+\sigma)\left[\frac{\delta}{m} - \frac{\beta}{k}\right]^{-1}\\ (\pi+\sigma+\rho)\cdot\frac{m}{\delta}\cdot\left[\frac{\delta}{m} - \frac{\beta}{k}\right] &= (i+\sigma)\end{aligned}$$

From this equation (17) and so the expression for $\hat{\imath}$ follows in a straightforward way.

E The zero lower bound on nominal interest rates

In short-run equilibrium the nominal interest rate is at the zero lower bound, $\hat{\imath} = 0$, when;

$$(\pi+\sigma+\rho)\left(\left(1 - \left(\frac{m}{k}\right)\left(\frac{\beta}{\delta}\right)\right)\right) = \sigma$$

The total differential of equation (21) with respect to σ and m yields

$$\left(1-\left(\frac{m}{k}\right)\left(\frac{\beta}{\delta}\right)\right)d\sigma-(\pi+\sigma+\rho)\left(\frac{1}{k}\right)\left(\frac{\beta}{\delta}\right)dm=d\sigma$$

which can be simplified to;

$$-\frac{d\sigma}{\sigma}\left(\frac{\sigma}{\pi+\rho+\sigma}\right)=\frac{dm}{m}\ \text{ or }\ \frac{d\sigma}{\sigma}=-\left(\frac{\pi+\rho+\sigma}{\sigma}\right)\frac{dm}{m}$$

This relationship upholds $\hat{\imath}=0$ when one of the policy instruments is changed. Thus, a one-percent increase in one instrument requires a corresponding percentage decrease in the other one. Now recall

$$\hat{c}_{|\sigma=0,\hat{\imath}=0}=\frac{(\bar{\pi}+\rho)\cdot k}{\beta}=\frac{(\bar{\pi}+\rho)\cdot m_0}{\delta}\ \text{ and }\ \hat{c}_{|\sigma>0,\hat{\imath}=0}=m_1\cdot\sigma\left[\delta-\frac{m_1}{k}\cdot\beta\right]^{-1}$$

where the indexation $m_i, i=0{,}1$ expresses the fact that m will be lower when $\sigma>0$, that is, $m_1<m_0$. I want to check whether $\hat{c}_{|\sigma=0,\hat{\imath}=0}\gtreqless\hat{c}_{|\sigma>0,\hat{\imath}=0}$. To this end let us suppose $\hat{c}_{|\sigma=0,\hat{\imath}=0}\leq\hat{c}_{|\sigma>0,\hat{\imath}=0}$. Then

$$\frac{(\bar{\pi}+\rho)k}{\beta}\leq m_1\cdot\sigma\left[\delta-\frac{m_1}{k}\cdot\beta\right]^{-1},$$

where, of course, $k=\bar{k}$. Then rearrangement implies;

$$\frac{(\bar{\pi}+\rho)k}{\beta}\cdot\left[\delta-\frac{m_1}{k}\cdot\beta\right]\leq m_1\cdot\sigma$$
$$\left(\frac{k}{m_1}\right)\left(\frac{\delta}{\beta}\right)\leq\frac{\pi+\rho+\sigma}{\pi+\rho}.$$

This inequality also holds when one takes logarithms. Thus, the claim would have to be that;

$$\ln\,k-\ln\,m_1+\ln\left(\frac{\delta}{\beta}\right)\leq\ln\,(\pi+\rho+\sigma)-\ln\,(\pi+\rho)$$

Taking the total differential of this expression yields that

$$-\frac{dm_1}{m_1}\leq\frac{d\sigma}{(\pi+\rho+\sigma)}=\frac{d\sigma}{\sigma}\left(\frac{\sigma}{\pi+\rho+\sigma}\right)$$

would have to hold. But as can be ascertained from above, both sides of this inequality are equal when $\hat{\imath} = 0$ is upheld. Hence, the introduction of money depreciation coupled with a lower money stock when $\hat{\imath} = 0$ does not bear on consumption in equilibrium when the economy's interest rate is at the zero lower bounds. This is the argument presented in the main text.

F Quotes [30]

- "The material part of the money has for economic life about the same importance that the leather of a football has for the players. The players do not concern themselves with the material of the ball, or with its ownership. Whether it is battered or dirty, new or old, matters little; so long as it can be seen, kicked or handled the game can proceed. It is the same with money. Our aim in life is an unceasing, restless struggle to possess it, not because we need the ball itself, the money-material, but because we know that others will strive to regain possession of it, and to do so must make sacrifices. In football, the sacrifices are hard knocks, in economic life they are wares, that is the only difference. Lovers of epigram may find pleasure in the following: **Money is the football of economic life.**" (Gesell (1920), p. 78.
- "Money requires the State, without a State money is not possible; indeed the foundation of the State may be said to date from the introduction of money. Money is the most natural and the most powerful cement of nations. The Roman Empire was held together more by the Roman currency than by the Roman legions. When the gold and silver mines became exhausted, and coins could no longer be struck, the Roman Empire fell asunder." (Gesell (1920), p. 81.
- (*"Usually when a German wants anything he also wants the opposite.", Bismarck. (Gesell (1920), p. 82.)
- "This revenue of the currency administration is an accidental by-product of the reform and is comparatively insignificant. The disposal of this revenue will be specially provided for by law. (*For other methods of applying the principle of Free-Money see page 245.) p.124"
- "In all conceivable circumstances, in fair weather and foul, demand will then exactly equal: - The quantity of money circulated and controlled by the State. Multiplied by: The maximum velocity of circulation possible with the existing commercial organization. What is the effect on economic life? The effect is that

[30] These quotes may or may not be included in a published version.

we now dominate the fluctuations of the market; that the Currency Office, by issuing and withdrawing money, is able to tune demand to the needs of the market; that demand is no longer controlled by the holders of money, by the fears of the middle classes, the gambling of speculators or the tone of the Stock Exchange, but that its amount is determined absolutely by the Currency Office. The Currency Office now creates demand, just as the State manufactures postage stamps, or as the workers create supply. When prices fall, the Currency Office creates money and puts it in circulation. And this money is demand, materialized demand. When prices rise the Currency Office destroys money, and what it destroys is demand. Thus, the Currency Office controls the tone of the market, and this means that we have at last overcome economic crises and unemployment. Without our consent, the price level can neither rise nor fall. Every movement up or down is a manifestation of the will of the Currency Office, for which it can be made responsible. Demand as an arbitrary act of the holders of money was bound to cause fluctuations in prices, periodic stagnation, unemployment, and fraud. Free-Money makes the price level dependent on the will of the Currency Office which uses its power, in accordance with the purpose of money, to prevent fluctuations. Confronted with the new money everyone will be forced to conclude that the traditional custom of storing up reserves of money must be abandoned since reserve money steadily depreciates. The new money, therefore, automatically dissolves all money hoards, those of the careful householder, of the merchant and of the usurer in ambush for his prey." p. 127.

- "Under Free-Money, when sales slacken and prices decline, the explanation is no longer given that too much work has been done, that there has been overproduction. We now say that there is a shortage of money, of demand. Whereupon the National Currency Office puts more money in circulation: and since money is now simply embodied demand, this forces prices up to their proper level. We work and bring our wares to market - that is supply. The National Currency Office then considers this supply and puts a corresponding quantity of money on the market - that is demand. Demand and supply are now products of labour. **There is now no trace of arbitrary action, desires, hopes, changing prospects, or speculation, left in demand.** We order just the amount of demand that we require, and just this amount is created. Our production, the supply of goods, is the order for demand, and the National Currency Office executes the order." p. 134

- "And Heaven help the controller of the Currency office if he neglects to do his duty! He cannot now, like the administration of the old Banks of Issue, entrench himself behind platitudes about having to satisfy "the needs of commerce". The duties imposed on the National Currency Office are sharply defined and the weapons with which we have equipped it are powerful. The German mark, formerly a vague, indefinite thing, has now become a fixed quantity, and for this quantity, the officials of the Currency Office are held responsible.

  We are no longer the sport of financiers, bankers, and adventurers; we are no longer reduced to waiting in helpless resignation, until, as the phrase used to be, "the state of the market" has been created and improved. We now control demand; for money, the supply of which is in our power, is demand - a fact which cannot be too often repeated or too strongly emphasized. We can now see, grasp and measure demand - just as we can see, grasp and measure supply. **Much produce - much money; less produce - less money. That is the rule of the National Currency Office, an astonishingly simple one!"** p. 134

- "Many of those who have learned to separate money from gold, who have renounced the heresy of "intrinsic value" and convinced themselves of the importance of stable prices will now be inclined to argue as follows: Why not simply manufacture paper-money and bring it into circulation as soon as supply has overtaken demand or, in other words, when prices begin to fall? And conversely: Why not withdraw paper money from circulation and burn it when demand begins to exceed supply, that is, when prices begin to rise? This is merely a question of quantity: a lithographic press and a fireplace put it in your power to adapt demand (money) so exactly to supply (the wares) that prices remain constant." p. 111

- "Freemoney is not redeemed by the Currency Office. Money will always be needed and used, so why should it ever be redeemed? The Currency Office is, however, bound to adapt the issue of money to the needs of the market in such a manner that the general level of prices remains stable. The Currency Office will therefore issue more money when the prices of goods tend to fall, and withdraw money when prices tend to rise; for general prices are exclusively determined by the amount of money offered for the existing stock of goods. And the nature of Free-Money ensures that all the money issued by the Currency Office is immediately offered in exchange for goods." p. 123

- "The masses of paper money hoarded by private individuals (all private fortunes would finally have assumed that form) might any day have been set in motion by some trivial event, and this money, being only redeemable in the market in exchange for goods, would suddenly have become an enormous mass of demand which the State would have been powerless to control by means of the bonds and long-term bills. " p. 156

- "The sale of the currency stamps creates a regular annual revenue for the Currency Office. This revenue of the currency administration is an accidental by-product of the reform and is comparatively insignificant. The disposal of this revenue will be specially provided for by law." p. 124
- "The money reform deprives the Banks of Issue of the privilege of issuing banknotes. Their place is taken by the National Currency Office which is entrusted with the task of satisfying the daily demand for money.
- The National Currency Office does not carry on banking business of any kind. It does not buy or sell bills of exchange, it does not classify business firms as first, second and third rate.
- To put free money in circulation all public treasuries are instructed to exchange, when requested to do so, the old national metal money or paper money for free money; one dollar (franc, or shilling) of free money being given for one dollar (franc, or shilling) of the old money.
- Anyone not consenting to this exchange may keep his gold. No one will compel him to exchange it; there will be no legal pressure; no force will be employed. The public is merely warned that after the lapse of a certain term (1,2 or 3 months), the metal money will be only metal and no longer money. If by that time anyone still possesses metal money he is free to sell it for Free Money to a dealer in precious metals, but he must bargain about the price. The only form of money recognised by the State will be free money. Gold, for the State, will be a mere commodity like wood, copper, silver, straw, paper or fish oil. And just as today taxes cannot be paid in wood, silver or straw, so gold will not be available for the purpose of paying taxes after expiration of the term for exchange." p. 124.
- "With Supplementary Free-Money the legal depreciation is compensated in each transaction by a supplementary payment by the holder of the note, as at present in many countries with the purchase tax (sales tax). Theoretically, the principle of free money could be applied by a continuous regular inflation of prices of 5% annually, with, to protect creditors, a corresponding modification of long-term money contracts. (For 18 years the continuous irregular inflation, without

modification of money contracts, practised by almost all countries, has realised one aim of free money: the elimination of depressions and unemployment - but at the expense of creditors, and with many grave economic disturbances). " p. 205, the translator (1958). [31]

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[31] Note that the "continuous regular inflation of prices of 5% annually" corresponds to Gesell's idea that the rate, at which money should depreciate annually, would be around 5%.

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