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Résumé de l'article

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Facilitating Productive Struggle in an Online Secondary Education Mathematics Methods Course: Experiences of Pre-Service Teachers

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Abstract

This research investigated preservice teachers' (PSTs) experiences with productive struggle in an online environment, and how it influenced their pedagogical knowledge. This action research study reports on secondary mathematics pre-service teachers (PSTs) who completed mathematical tasks in weekly synchronous Zoom sessions to develop their understanding of productive struggle. Vygotsky's (1978) Social Constructivist Theory and the growth mindset framework (Boaler, 2016; Dweck 2006; 2008) provided the lenses through which the research was designed and analyzed. Findings show that presenting opportunities for PSTs to experience productive struggle informs their pedagogical practice, and that providing this experience in an online environment is possible. PSTs gained an increased understanding of the importance of collaborative work, asking purposeful questions, and the alignment between productive struggle and improved mathematical understanding.

Introduction

The National Council for Teachers of Mathematics proposes that students must confront and work through their mathematical "uncertainties" during class activities (NCTM, 2014). That is, they must participate in productive struggle. Productive struggle is defined as "a learning opportunity that students participate in when required to apply prior knowledge to challenging tasks, learning from their mistakes in the process" (Casler-Failing et al., 2022, para. 1). This productive struggle advocacy was built upon Hiebert and Grouws' (2007) argument that struggle was necessary to develop conceptual understanding of mathematical concepts. For pre-service teachers (PSTs) to incorporate productive struggle into their classrooms, they must first experience what it feels like to struggle. PSTs need opportunities to productively struggle with mathematical concepts to support their future students' development of critical thinking, problem-solving, and collaborative skills. Productive struggle is not only essential to doing mathematics but also for teaching mathematics.

Previous research indicates that when incorporating productive struggle into K-12 mathematics classes, it is important to not only consider students' needs, backgrounds, and experiences (Ewing et al., 2019; Townsend et al., 2018) but also the teacher's ability to prepare and conduct the lesson (Townsend et al., 2018), along with the rigour of the tasks provided (Ewing et al., 2019; Livy et al., 2018). PSTs must understand the benefits of making mistakes when designing and implementing purposefully chosen tasks that elicit productive struggle. Their understanding of the power of mistakes is necessary to develop their future students' ability "to perceive mathematics as both useful and worthwhile" (National Research Council, 2001, p. 131), and to support students as they work through their own mistakes along the path to successful solution strategies.

Existing research speaks to the importance of productive struggle in synchronous and in-person classes (e.g., Casler-Failing & Collins, 2022; Ewing et al. 2019; Livy et al., 2018; Warshauer, 2014), and the importance of providing PSTs with these experiences to inform their future practice (e.g., Russo et al., 2021; Valentine & Bolyard, 2018; Zeybek, 2016). However, the examination of productive struggle when learning mathematics in an online format has yet to be closely studied. The logistics of facilitating a course across two campuses and the COVID-19 pandemic moved the instruction of my mathematics methods' course online, presenting an opportunity to investigate this topic in an online environment. This research aimed to investigate how secondary mathematics PSTs' experiences with productive struggle in their online methods course informed their understanding of the benefits of incorporating productive struggle to promote student learning. The research questions guiding this study were:

1. When provided challenging mathematical tasks, how do pre-service teachers classify their productive struggle experience in an online methods' course?
2. How do the productive struggle experiences support pre-service teachers' development in understanding how to incorporate productive struggle to support student learning in grades 6-12?

Theoretical Perspective

The research design in this manuscript was based on Vygotsky's (1978) Social Constructivist Theory and the growth mindset framework (Boaler, 2016; Dweck, 2006; 2008). These were chosen as the lenses through which this research was developed and analyzed, due to the way the process of learning is supported when these contexts are present. The connection of these perspectives to this research, as well as their interconnectedness, are described below.

Vygotsky (1978) proposed that learning does not occur in a vacuum; it is inherently a social activity that flourishes and deepens when individuals can collaborate. Through discourse and scaffolding, students can expand their Zone of Proximal Development (ZPD) to solve problems collaboratively that may seem impossible to complete independently. ZPD, as defined by Vygotsky (1978), is "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers" (p. 86). This idea cannot be thought of as "solely a characteristic of the child or of the teaching, but of the child engaged in collaborative activity within specific social (discourse) environments" (Moll & Whitmore, 1993, p. 20). Based on this understanding of ZPD, the discourse provided by peers within and among collaborative groups, and the scaffolding provided through purposeful questioning by the teacher, can support all learners through productive struggle by providing an avenue to think through the problems posed. The obstacles faced, during the problem-solving process, no longer create unproductive struggle when the process becomes collaborative and incorporates social discourse.

Individuals can overcome productive struggle during peer collaboration, if they persevere and strategically use tools (or resources). This is most evident when they possess a growth mindset (Dweck, 2006; 2008). When this happens, they acknowledge the benefit of working hard and not giving up. They realize their ability is connected to the effort they extend. For example, athletes, actors, and video gamers understand that practice and effort will improve their performance and/or product. This notion that effort produces results was connected to the learning of mathematics (Boaler, 2016) to break the ideology that being bad at math is irreversible. Possessing a growth mindset can support one’s ability to persevere through difficult tasks when collaborating, as they productively struggle through a problem. A significant action in supporting the development of a growth mindset is providing praise for effort, not results (Boaler, 2016). This practice was modelled extensively by the researcher throughout this study with PSTs.

Merging the two concepts of Social Constructivist Theory and the growth mindset was an important facet of this research. When developing the activities for this research, it was understood that it was important to create opportunities for discourse and collaboration. In addition, PSTs needed to possess and understand the necessity of a growth mindset to support their learning and participation in the collaborative discourse.

Literature Review

In this section, the research on productive struggle, online learning, and teaching mathematics in an online environment will be reviewed. Each of these research areas was instrumental in developing this research study.

Productive struggle

The research reported in this study is based on the premise that PSTs can only implement productive struggle in their classrooms if they understand its importance in mathematical learning. Much research has been conducted that reports on the significance of PSTs participating in learning experiences to enhance their pedagogical skills that support deeper student learning (e.g., Kolb & Fry, 1975). If PSTs do not know what it feels like to experience productive struggle, they may be unable to support it as a pedagogical practice. While the number of studies is limited regarding this investigation of productive struggle with PSTs, the results were beneficial for informing the research in this manuscript.

Valentine and Bolyard (2018) found that elementary PSTs focused on supporting students to help them complete challenging mathematical tasks, rather than focusing on the importance of making mistakes as opportunities to learn (Kapur, 2014). After receiving instruction on problem-based approaches, Russo et al. (2021) found that elementary teachers had a more positive outlook on the benefits of struggle in the learning process. Valentine and Bolyard’s (2018) study concluded that PSTs need to understand the benefits of productive struggle more clearly in order to support their students through the struggle.

It is not enough, however, for PSTs to experience the struggle; they must also understand “the developmental progression in their thinking and learning” (Zeybek, 2016, p. 411). PSTs must understand the different stages of struggle (Warshauer, 2014), and how they can be connected to mathematical proficiency (National Research Council 2001), or lack thereof. The chosen problems, or tasks, must be purposefully chosen to engage the students, must be culturally relevant for the students and PSTs (Ewing et al., 2019; Townsend et al., 2018), and must provide adequate amounts of rigour (Ewing et al., 2019; Livy et al., 2018). The problems should not be so easy that no struggle is necessary, but they also should not be so difficult that a solution is impossible at the individual's level of learning. Choosing purposeful mathematical problems is necessary to model for PSTs, because they must understand how to prepare and conduct lessons that incorporate similar types of struggle for their students (Townsend et al., 2018). PSTs must see themselves as facilitators of productive struggle, based on their personal experiences.

Casler-Failing and Collins (2022) researched in a middle-grade math methods course as PSTs learned about LEGO robotics. The research was conducted to support their understanding of how robotics can be used as an instructional tool to teach and apply mathematical concepts. As PSTs built, learned how to program, and created mathematics lessons with the robotics, they participated in productive struggle. The study found that PSTs reported they understood how productive struggle supported learning, and that they overcame their struggles through perseverance and collaboration with peers.

Online learning

Today, more classes are moving to online formats. Much of this movement has been in higher education, and has been in development since the early 2000s to support adult learners who are trying to balance work, home, and collegiate life. In the past five to ten years, there has also been a shift at the K-12 level of education, which has been much more prevalent since the COVID-19 pandemic. This shift to online instruction necessitates a change in pedagogical practices. What works in a face-to-face environment is not always applicable to an online learning environment, whether synchronous or asynchronous.

Creating collaborative learning spaces is one area that requires planning when teaching in an online environment. The decisions will vary, based on the form of learning - synchronous or asynchronous (Razmerita et al., 2020). Group work can be especially difficult for online learners when collaborators cannot meet in person or via a digital platform (Razmerita et al., 2020). Creating opportunities for engagement between students and among students and teachers is considered a necessary component of online instruction (Cerezo et al., 2016). In addition to collaborative activities, it is necessary to incorporate meaningful instructional practices that encourage learners to engage with the material (Taghizade et al., 2020), while also creating meaningful assignments with clear instructions (Shea & Bidjerano, 2008) that support the success of all students. Branon and Essex (2001) report that synchronous sessions that promote brainstorming and decision-making benefit student success when they are required to participate in problem-solving tasks. What should be apparent in the research is that it does not matter which form the online learning takes, synchronous or asynchronous, as it is arduous work to modify most face-to-face activities for an online environment without losing content knowledge and interaction (Shuey, 2002).

Teaching mathematics in an online environment

Many mathematics teachers have incorporated online activities into face-to-face learning through apps (e.g., Desmos, Geogebra, Cool Math Games), and participated in a form of online learning via flipped classrooms for almost two decades. Recently, the COVID-19 pandemic mandated that all teachers move all instruction to an online format – a task that was likely overwhelming for many teachers, but not for all. Research has found that students can continue to participate in productive discourse (Harbour & Denham, 2021) and productive struggle opportunities (Russo et al., 2021), while continuing to grow in their mathematical understanding (Spitzer & Musslick, 2021) in an online environment.

Harbor and Denham (2021) propose that integrating the "The Five Practices," as proposed by Smith and Stein (2018), into an online learning environment can promote discourse and student learning, if the online design includes an opportunity for asynchronous individual work in both whole group and small group discourse, conducted synchronously. Russo et al. (2021) found that although primary teachers in Australia could provide productive struggle opportunities online, it was much more difficult than implementing them in a face-to-face classroom. The difficulty of incorporating opportunities for productive struggle for the five-to-eight year-old students was reported to be negative parental attitudes regarding the practice of struggle, lack of teacher support during the struggle, lack of peer interactions, and an inability for students to access materials (Russo et al., 2021). Although research reports on the difficulties of teaching online, there are also successes, as presented by Spitzer and Musslick (2021) with their study in Germany.

Spitzer and Musslick (2021) investigated the effects of *The Bettermarks* software used by German schools in grades four to ten. The research compared the mathematical gains for students who used the software before school shutdowns (March to June 2019) to students who used the software before and during the shutdowns (March to June 2020). The results show that students who utilized the software during the school shutdowns improved their mathematical performance relative to their performance before the shutdowns. These findings were reinforced by teachers who reported the need to assign higher-level problems during the shutdown in order to maintain the rigour of the tasks (Spitzer & Musslick, 2021). Another finding was that the performance gap was narrowed during the school shutdown. Lower-performing students achieved more gains than higher-performing students (Spitzer & Musslick, 2021).

The existing literature in this section provided a foundation for developing the research reported in this manuscript. The literature documents the importance of PSTs experiencing productive struggle to support their future pedagogical practice, ensuring high-leverage practices in online environments include opportunities for synchronous instruction, and evidence that mathematics instruction can promote student learning in an online environment.

Methodology

This research sought to investigate how secondary education PSTs' experiences with productive struggle in a fully online environment informed their pedagogical knowledge and strategies for supporting their future students. Clearance was obtained from the university's Institutional Review Board for this research. A qualitatively oriented mixed-methods framework (Morse & Cheek, 2015) incorporating action research (Anderson & Herr, 2005) was used for data collection in a secondary mathematics methods course. The action research design was chosen because it creates "knowledge that is useful, valid, descriptive…, and informative" (Argyris et al., as cited in Anderson & Herr, 2005, p. 14). Although other frameworks could have been utilized for this research (i.e., case study, phenomenology), action research allowed for revision in the activities and data collection process that a more stringent research design would prevent. Vygotsky's (1978) Social Constructivist Theory and the growth mindset framework (Boaler, 2016; Dweck, 2006; 2008) provided the lenses through which this research was designed and analyzed.

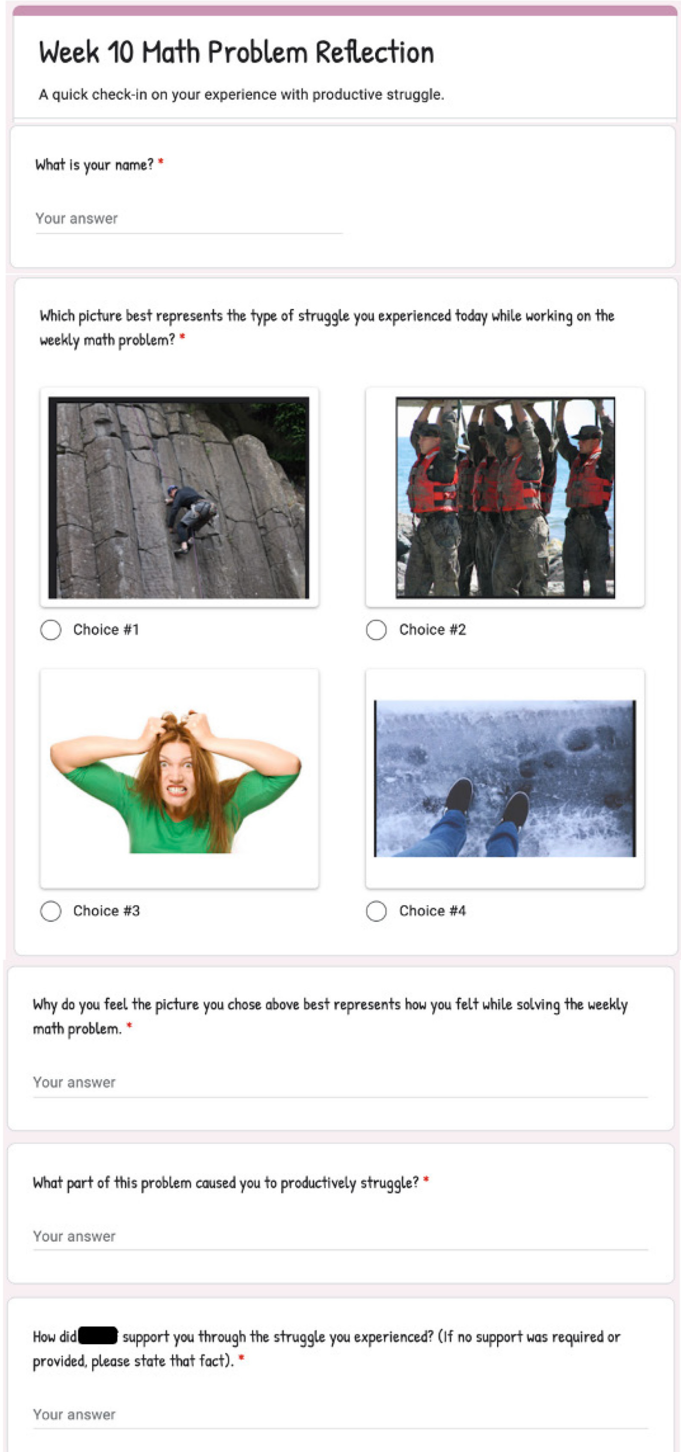
Setting and participants

This research was conducted at a large, rural university in the southeastern part of the United States. The participants were first-semester seniors enrolled in an undergraduate secondary education program in the College of Education. The five secondary mathematics education majors (three identified as female and two identified as male) were all enrolled in the secondary mathematics methods' course during the fall of 2020. They participated in weekly, synchronous Zoom sessions. The author served the dual role of course instructor and sole investigator of this research study.

Curriculum

The PSTs met with the instructor once per week, for approximately one hour per Zoom session. During 12 of the 15 weekly sessions, a mathematical problem was posted to the PSTs that were retrieved from various sources, such as "Problems to Ponder" in *Mathematics Teaching in the Middle School* or "Calendar Problems" from *The Mathematics Teacher*, both journals formerly published by the National Council of Teachers of Mathematics (NCTM). Problem #4 was discussed across two consecutive weeks. Therefore, 11 problems were investigated during the semester (see Appendix for all problems posed). PSTs were randomly separated into groups of two to three in Zoom break-out rooms to work through the problem each week and develop a solution. The course instructor switched between rooms to assess progress and pose questions to scaffold PSTs' solution processes. Once answers had been developed in both groups, they would regroup as a whole class. Each group would share their solution strategies verbally, and, when possible, visually (often by holding up their paper to the camera).

After each class, PSTs were required to submit a written description of their solution process and complete a brief survey (see Figure 2) describing the productive struggle they experienced, if any. This weekly survey design was based upon the suggestions presented in *Productive math struggle: A 6-point action plan for fostering perseverance* (SanGiovanni et al., 2020), which provides practical strategies for incorporating productive struggle into classroom activities, along with strategies for students to reflect on their experiences while working through the struggle. The pictures were chosen based on the reading by SanGiovanni et al. (2020). They were intended to reflect slow and steady progress (choice #1), the benefit of teamwork (choice #2), frustration (choice #3), and risk-taking/careful progression (choice #4). These pictures allowed students to report how they felt during the problem-solving process each week. The first three weeks of survey completion only included the first three questions (name, picture choice, and justification of chosen picture). The last two questions were added in response to the instructor's reflection on the survey data each week in order to obtain richer data regarding the productive struggle and instructor support from the perspective of the PSTs. Action research allows a recursive process in developing and collecting research data, which supported this decision (Anderson & Herr, 2005).



Week 10 Math Problem Reflection

A quick check-in on your experience with productive struggle.

What is your name? *

Your answer

Which picture best represents the type of struggle you experienced today while working on the weekly math problem? *

Choice #1

Choice #2

Choice #3

Choice #4

Why do you feel the picture you chose above best represents how you felt while solving the weekly math problem. *

Your answer

What part of this problem caused you to productively struggle? *

Your answer

How did [redacted] support you through the struggle you experienced? (If no support was required or provided, please state that fact). *

Your answer

Figure1: Survey completed by PSTs after each online session.

Data collection and analysis

In addition to the problem and survey completed each week by the PSTs, an entrance and exit survey was conducted during the first and last weeks of class to evaluate the PSTs' growth, or lack thereof, in understanding the benefits of productive struggle as an instructional strategy. Observational notes were collected during each Zoom session to document struggle and PST comments. The entrance and exit survey questions related to PSTs' experiences with, and understanding of, productive struggle. Both Likert scale and open-response questions were included in both surveys.

Each data source was analyzed holistically to determine emergent themes (e.g., frustration, asking questions, perseverance; Creswell, 1998), and then more thoroughly, to determine overarching themes (e.g., the benefit of collaboration, the importance of productive struggle in the learning process) through an iterative process. The PSTs' responses to the final question on the weekly survey, referencing instructor support and the observational notes, were also independently evaluated for emergent themes (e.g., questioning, encouragement), before developing overarching themes. After determining overarching themes for each data source, the themes were reviewed across the data sources to support construct validity (Patton, 2002; Yin, 2018). Every precaution was taken to reduce implicit researcher bias, by conversing with critical friends throughout the data-analysis process. These critical friends challenged the findings by asking probing questions, and offering alternative perspectives, by examining the data through alternative lenses (Costa & Kallick, 1993).

Findings

After one semester, the findings provide evidence that presenting opportunities for PSTs to experience productive struggle informs their pedagogical practice. Furthermore, it is possible for methods instructors to provide this experience in an online environment.

Types of productive struggle experienced by PSTs

Understanding how PSTs felt during this struggle was an important component of the research. To reflect on how they progressed through the struggle, PSTs were asked to choose the picture (see Figure 1) that best represented their experience. This data is shown in Figure 2.

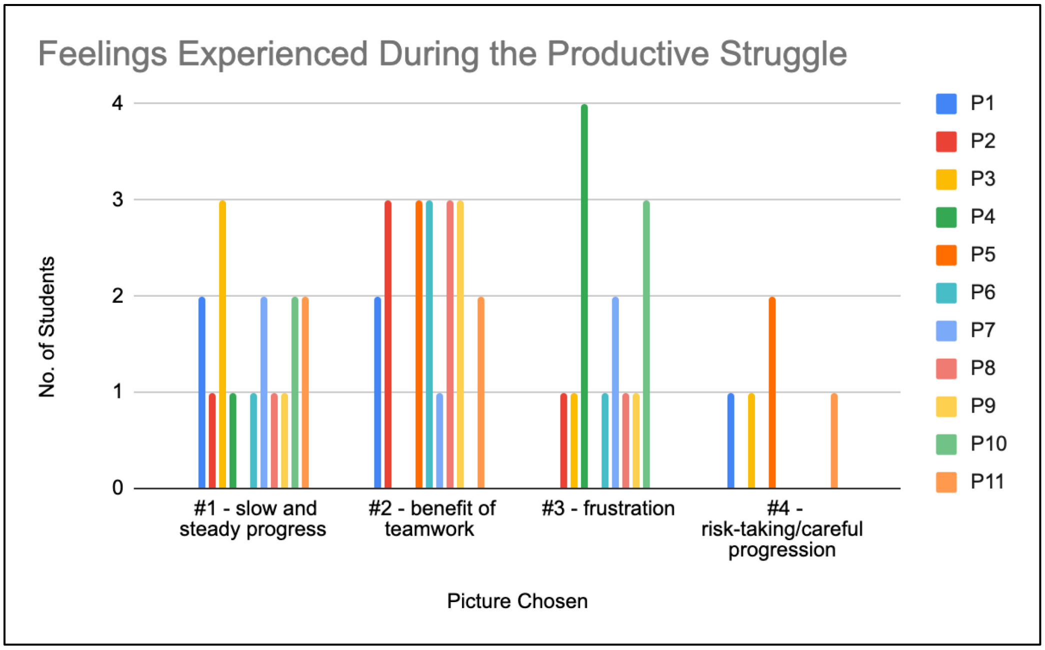


Figure 2: Feelings experienced by PSTs throughout the semester.

PSTs most often experienced slow and steady progress and the benefit of teamwork. These themes were also present in the survey responses and the observational notes, as will be discussed later in this manuscript. Frustration appeared to be most dominant in problems #4 and #10. These problems dealt with determining the total number of triangles that can be formed on a geoboard, and a logic-type problem providing four statements from which a final solution can be determined, respectively (see Appendix). Regarding problem #4, of the four PSTs who regarded their struggle as causing frustration, three also reported appreciating the challenge it presented, because it required attention to detail. With problem #10, all three PSTs became frustrated when they did not know how to begin the problem. They referred to the ambiguity in the statements as the primary reason for their frustration, and did not report being receptive to the challenge presented by this problem.

Influence of PSTs' productive struggle to inform future practice

PSTs' understanding of productive struggle, from a student's perspective, deepened throughout the course, when they experienced how it could support learning. This development is evidenced by comparing their entrance and exit survey data regarding their definition of productive struggle (see Table 1).

Table 1: Evidence of PSTs' development in their understanding of productive struggle.

Entrance Survey Definition	Exit Survey Definition
allowing students to work through difficulties in a way that is beneficial to their learning	work[ing] through difficulties in a way that builds understanding and causes growth

process of facing resistance while learning in order to produce a better outcome of learning	use of difficult concepts and strategies to solve problems; the result of providing the difficulty is to give a new perspective and to learn
the idea that students learn and grow when they are presented with a problem that they don't immediately know how to fix or solve, but eventually get to a solution through trial and error (whether its mentally or physically), instead of being given the answer	the gray area between the frustration of not knowing how to reach the desired result and finding the solution; the point where you don't immediately know the answer, but you experience a struggle that helps you learn and grow as you figure out the process to get to the solution
difficulty of generating concrete ideas and thinking creatively in order to solve the problem at hand	the perseverance and focus required to contemplate and fully understand a concept; with this struggle comes deep content knowledge and understanding
where teachers/facilitators purposely allow students/people to struggle, with the intent of yielding a benefit in the end (with regard to learning)	a pedagogical strategy that allows students to create their own conjectures and conclusions about a task/concept, without hints or "give-aways" from the teacher; [requires] students to self-assess their own thinking and gives them autonomy over their learning

Four of the five PSTs approached this definition on the entrance survey from a student perspective, but the fifth response speaks to the teacher's role in "purposely allow[ing]…struggle." The exit survey responses also focus on the student, but appear to do so by positioning the student as a doer and learner in the process (e.g., building understanding and providing new perspectives). The fifth response addresses the teacher's role, specifically referring to productive struggle as a "pedagogical strategy," and adds the importance of student autonomy. Another common theme within the exit survey definitions is the connection to learning and understanding. Reading through these responses, there are references made to building understanding, providing new perspectives, learning and growing, deep content knowledge, and student self-assessment. Each phrase represents growth in PSTs' understanding of the benefits of productive struggle from a pedagogical viewpoint. Phrases such as "beneficial to…learning" and "difficulty of generating concrete ideas" are replaced with "builds understanding" and "perseverance and focus," thus reflecting development in their pedagogical vocabulary and classroom expectations.

The exit surveys specifically asked, "How will your understanding of productive struggle support your pedagogical practices when working with secondary mathematics students?" Responses to this question illuminated how the experience of participating in productive struggle, throughout the semester, informed their pedagogical knowledge. PSTs acknowledged the importance of possessing knowledge of alternative strategies to support the productive struggle. They developed an understanding that "everyone's journey with mathematics is different" (Charlotte; all student names are pseudonyms), and that opportunities must be created to allow learners to collaborate and share their strategies. An important realization was the need to anticipate where unproductive struggle may occur in a lesson, so that the teacher can prepare questions and plan supports to move students toward productive struggle.

The weekly surveys and exit survey data reflected the PSTs increased understanding of the importance of collaboration while engaging in productive struggle. This was evidenced through such comments as, "The opportunity to experience productive struggle with my classmates and see how it helped us work as a team was definitely a positive experience" (Emma, exit survey). Additional evidence was provided in comments from the weekly surveys, such as, "We worked as a group, we were able to help each other and come to a solution" (Sierra, problem-one survey), and "We all thought and talked logically about the task…before we attempted it" (Emma, problem-two survey).

Understanding the importance of productive struggle, classifying their feelings associated with struggle, and learning how to persevere through it was an important aspect of the research from a learner's perspective. However, it was also necessary to model how to support productive struggle from a teacher's perspective. To evaluate PSTs' reliance on the instructor while solving problems (and reflecting on the process), the weekly surveys asked PSTs to share how they felt supported through the struggle. The cross-cutting themes found in the responses and observations were: clarification of the problem, encouragement, additional time, probing questions, and validation of thinking. When reflecting on the instructional strategies utilized to support productive struggle throughout the semester, Charlotte stated, "The actual solutions were of lesser importance, so it reduced the level of frustration I felt, and allowed me to focus on the journey" (problem #4 survey). Emma shared, "In many of my math classes, I was just given the solution or graded for a correct or incorrect answer, rather than having the chance to slowly work through and figure it out on my own. This helped me [to] see that I really learn much more, and have a more permanent understanding of the material, when I work through that productive struggle without the fear of an incorrect answer" (exit survey). Noah claimed that "[t]hrough productive struggle students can see that the learning process isn't [about] getting everything right all the time, but rather [that] mistakes will happen, but you learn from them" (exit survey). These responses provide evidence of the learning developed by PSTs regarding the instructional strategies they experienced as a learner. These experiences will carry into their instructional practices. Overall, the positive effect of asking purposeful questions to support learning and struggle, rather than guiding the learning to a specific outcome, created a deeper understanding of the alignment between productive struggle and PSTs' ability to think critically and problem-solve.

Discussion

This research investigated the following questions: When provided challenging mathematical tasks, how do PSTs classify their productive struggle experience in an online methods course? How do the productive struggle experiences support PSTs' development in understanding how to incorporate productive struggle to support student learning in grades 6-12?

Concerning the first question, the data shows that PSTs classified their productive struggle experiences as being influenced by teamwork, and determining solutions through a slow and steady process. Although frustration did occur, primarily in response to problems #4 and #10, PSTs still reported respect for the challenge provided. Of the 55 total weekly responses requiring PSTs to classify their productive struggle experience, 36 responses were linked to the "benefit of teamwork" and "slow and steady process," representing 65% of the responses. It was important for PSTs to position themselves as a student learner to understand how to best support their future students (Zeybek, 2016).

When evaluating the observational notes and weekly student surveys, it appears that the online format of the class presented only some of the productive struggles experienced by the PSTs. They could collaborate among their peer groups in the breakout rooms to support one another, as they would in a traditional face-to-face classroom environment, when provided the opportunity to collaborate. The social interactions leading to the development of understanding (Vygotsky, 1978) did not appear to be influenced by the online environment. Additionally, these reported experiences reflect the importance of possessing a growth mindset (Boaler, 2016; Dweck 2006; 2008) and participating in collaborative learning (Vygotsky, 1978).

The second question was answered via the PSTs' responses to the weekly surveys and exit survey, as well as observational notes. This data provided evidence that the opportunities to participate in productive struggle allowed PSTs to develop a strong understanding of its benefits as it relates to student learning. Such statements as, "I will be able to anticipate struggle and properly guide my students to the solution, not through providing answers, but by supporting [their] lack of understanding and providing hints and suggestions, in order to create solid mathematicians" (Liam, exit survey) reflect PST development. PSTs also understood how one's mindset (Dweck, 2006, 2008) could support or hinder their ability to persevere through the struggle, as reflected in Emma's comment, "…there were definitely times where I was experiencing frustration more than productive struggle. It completely depended on the mindset I was in, if I was more tired than usual, etc." (exit survey). Understanding the importance of productive struggle, classifying their feelings associated with it, and learning how to persevere through the struggle was an important aspect of the research from a learner's perspective, which reflects their productive disposition (National Research Council, 2001) and growth mindset (Boaler, 2016; Dweck, 2006; 2008).

This research is important to teacher education, and mathematics research in particular, due to its alignment with several *Standards for Preparing Teachers of Mathematics* (AMTE, 2020). This research created opportunities for PSTs to experience productive struggle as "student … learners of mathematics" (Standard C.3; AMTE, 2020, p. 18), to develop an understanding of the role productive struggle can play in the "social contexts of mathematics teaching and learning" (Standard C.4; AMTE, 2020, p. 21), and created "opportunities to learn to teach mathematics" (Standard P.3; AMTE, 2020, p. 33). Additionally, this research reflects the findings of Kolb and Fry (1975) that PSTs develop more robust pedagogical knowledge from participating in experiential learning. Noah reported that the productive struggle experienced each week "... allowed me to discover ... which ways I prefer to solve problems" (exit survey). Sierra found that she "liked getting to understand different ways of understanding problems by hearing my classmates' interpretations" (exit survey), which provides evidence of the importance of giving all learners a voice in the classroom, and how the social aspect of learning can promote understanding (Vygotsky, 1978). Emma realized that "there are outside factors you have to consider when trying to support your students through productive struggle…. Sometimes your students may not be in the right mindset, or have the energy to learn that way, so you have to prepare alternative ways to guide and teach them" (exit survey). Additionally, through this experience, PSTs have gained empathy. They understand how it feels to struggle and through instructor modelling, have learned how to support their future students through productive struggle.

Implications and Limitations

Working through problems online takes much more time than in a face-to-face environment. Several of the problems used in this class were also completed in a face-to-face middle-grade methods' class (designed for PSTs planning to teach grades 4-8), so an objective time comparison could be made. The problems took twice as long, minimally, to complete in the online environment. The additional time commitment was partially due to the PSTs being split among different break-out rooms, and the instructor having to move between online rooms, rather than circulate about a physical room reviewing student work. This leads to a second limitation: the PSTs' solution processes as they worked could not be viewed by the instructor. PSTs were asked to hold up their papers to the camera, but it was still difficult to see. This limitation required the instructor to be careful with questioning, so that strategies or hints did not remove the struggle from the problem. A third limitation is the lack of hands-on materials available during the solution process, an obstacle also noted in the research of Russo et al. (2021). In a middle-grade methods' class, there are various manipulatives available for PSTs to use as they work through the problems. However, in a virtual environment, manipulatives are limited to what the PSTs have immediately available (physical or virtual). For instance, problem #2 asked PSTs to move matchsticks (see Appendix). Some PSTs used popsicle sticks or pens/pencils that they had readily available. However, based on PSTs' weekly reflections, the struggle that they experienced was solely due to the math problems presented, and not the online environment (apart from access to manipulatives).

Conclusion

Although limited in scope, due to the small number of participants, this research provides valuable insights into the importance of PSTs experiencing productive struggle – whether in a face-to-face or online setting. Most importantly, this research adds to the current body of research on productive struggle, by providing evidence that it can be experienced by PSTs and that instructional strategies for supporting the struggle can be modeled in a fully online environment. The data shows PSTs found the productive struggle experiences beneficial to understanding how they could incorporate similar activities that would support their future students' development of mathematical thinking, problem-solving, and collaborative skills. This research supports previous findings (e.g., Casler-Failing & Collins, 2022; Russo et al., 2021). An additional important outcome of this research, as reported by Charlotte, is that this experience "helped strengthen [her] mathematical identity and confidence" (exit survey). Future studies are planned to continue this research, with additional secondary mathematics' methods courses in a fully online environment.

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Appendix

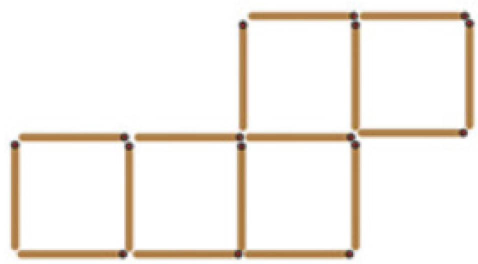
Problem #1: Golden Apples

Source:
Mayer, C. & Sallee, T. (1983). *Make it simpler: A practical guide to problem solving in mathematics*. Addison-Wesley.

A prince picked a basketful of golden apples in the enchanted orchard. On his way home, he was stopped by a troll who guarded the orchard. The troll demanded payment of one-half of the apples plus two more. The prince gave him the apples and set off again. A little further on, he was stopped by a second troll guard. This troll demanded payment of one-half of the apples the prince now had plus two more. The prince paid him and set off again. Just before leaving the enchanted orchard, a third troll stopped him and demanded one-half of his remaining apples plus two more. The prince paid him and sadly went home. He has only two golden apples left. How many apples had he picked?

Problem #2: Moving Matches
Source:
NCTM. (2019). May calendar of problems. *Mathematics Teacher, 112*(7), 520-525.

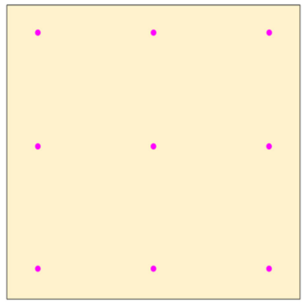
What is the minimum number of matches you can move (not remove) to create 4 congruent squares instead of 5 congruent squares? An overlapped or incomplete square is not permitted.



Problem #3: Weight of Water
Source:
NCTM. (2019). May calendar of problems. *Mathematics Teacher, 112*(7), 520-525.

A jar consisting of water and sand weighs 10 ounces; the water is 90% of the weight. After some of the water is poured out, the water is only 80% of the weight. What is the weight of the water still contained in the jar?

Problem #4: Triangles on a Geoboard (discussed for 2 classes)
Source:
Perkins, A. & Pettis, C. (2019, January). Triangles on a geoboard. *Mathematics Teaching in the Middle School, 24*(4), 197.



Choose three points on the grid to create a triangle. (Note: all vertices must correspond with the points on the geoboard grid for the problems below).

A. How many triangles is it possible to create on this geoboard? Explain.
B. How many different triangle areas are possible on a 3x3 geoboard? Explain.
C. Is it possible that a larger geoboard would allow you to make a triangle with a smaller area than you found in #2? Explain.

Problem #5: What's the Combined Distance?
Source:
NCTM. (2019). May calendar of problems. *Mathematics Teacher, 112*(7), 520-525.

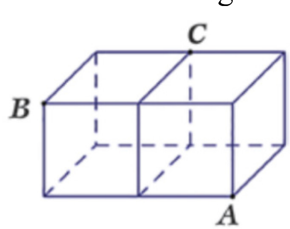
On a number line, C is twice as far from A as it is from B. If AB = 7.5 in., find the number of inches between the two possible locations for C.

Problem #6: What's the Score?
Source:
NCTM. (2019). May calendar of problems. *Mathematics Teacher, 112*(7), 520-525.

Five students on a chess team play a game against each of four teammates. A win = 3 points, a loss = 0 points, and a draw = 1 point. After 10 games, the total scores for 4 students are 1, 2, 5, and 7. Find the 5th player's score.

Problem #7: What's the Area?
Source:
NCTM. (2019). April calendar of problems. *Mathematics Teacher, 112*(6), 440-446.

Two-unit cubes are placed side by side to create a rectangular prism. Point C, a unit cube vertex, is now the midpoint of an edge. Find the area of triangle ABC.



Problem #8: How Far Can the Witch Fly?
Source:
NCTM. (2018). October calendar of problems. *Mathematics Teacher, 112*(2), 120-125.

From sunset to sunrise, a witch has only 13 hours to fly. How far can she fly on her broomstick at 35 mph, if she flies at 39mph per hour on the return trip home?

Problem #9: Can You Prove It?
Source:
NCTM. (2018). October calendar of problems. *Mathematics Teacher, 112*(2), 120-125.

Prove that the three expressions 2k+1, 2k2+2k, and 2k2+2k+1will always generate a Pythagorean triple for any integer *k*. Then explain why the generated triple must be a *primitive* triple. (The integers in a primitive triple are relatively prime).

Problem #10: Consumer Interview
Source: Unknown

A certain product is sold as either a liquid or a powder. Consumers were interviewed, and a survey revealed that:

- 1/5 do not use the product,
- 1/3 do not use the powder form,
- 427 use both the liquid and powder form, and
- 2/7 do not use the liquid form.

What is the total number of customers interviewed?

Problem #11: Spiders and Webs
Source: Unknown

It takes 26 spiders to spin 26 webs in 26 minutes. At this rate, how long will it take 13 spiders to spin 78 webs?