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Some Considerations about Number

The word " number " is used in many ways. As G. Birkhoff says :

> Number means a positive integer such as 17, a real quantity such as π or -2, or an element of any of various abstract mathematical generalizations of the system of real numbers. These generalizations include complex numbers, quaternions and other hypercomplex numbers, modular numbers and transfinite cardinal and ordinal numbers . . .[1]

This multiple use of the word is also seen in Russell's definition of " number " as " anything which is the number of some class." If number did not refer to something different in the definition than it does in the definitum, the definition would be circular and thus useless. Russell is aware of this and says :

> Such a definition has a verbal appearance of being circular, but in fact it is not. We define ' the number of a given class ' without using the notion of number in general ; therefore we may define number in general in terms of ' the number of a given class ' without committing any logical error.[2]

Russell would call this multiple use of word " systematic ambiguity." It has been suggested that " analogous name " might be a better expression for such employment of a word.[3] However, not every word employed with different applications to many things is to be called " analogous." Some, such as the word " bark " used to signify the sound made by a dog and also the covering of the trunk of a tree, seem to have nothing in common other than the purely material elements of the language itself. Others, such as " healthy " when used to signify the body and food, are usually employed deliberately to indicate that the things thus commonly named have a certain common aspect which can be signified in this manner even though the things themselves possess quite different natures. In this context the word " number " as used by Birkhoff and by Russell would seem to be an analogous name. Supposing this to be true, there is yet another difficulty. The analogous name must signify only one common aspect in the various things signified by that name, unless we are to impose

1. Article on *Number*, Encyclopaedia Britannica, Ed. 1956.

2. B. Russell, "Definition of Number," in *The World of Mathematics*, edited by James R. Newman, Simon and Schuster, New York, 1956, p.542.

3. Charles De Koninck, "Metaphysics and the Interpretation of Words," in *Laval théologique et philosophique*, 1961, Vol. XVII, n.1, p.22.

analogy on analogy to the confusion of all.[1] What will this common aspect be in the analogous use of the word " number "?

Ernst Cassirer, in his book *Substance and Function*, makes a thorough study of the various theories put forth to explain how number is derived, in the chapter entitled " the Concept of Number." He comes to the conclusion that " the concept of number can, in fact, be nothing in any form of deduction but a pure relational concept.[2] Another expression of this idea is to be found in Boyer's history of the Calculus, where he says that, " From the definitions of number given above, we see that it is not magnitude which is basic, but order." In explaining this " order " he says, " The *essential* characteristic of the number two is not its magnitude, but its place in the ordered aggregate of real numbers. The derivative and the integral, although defined as limits of characteristic quotients and sums respectively, have, as a result, ultimately become, through the definition of number and limit, not quantitative but ordinal concepts. The calculus is not a branch of the science of quantity, but of the *logic of relations*." [3] A third instance showing that modern mathematicians conceive of relation as something essential in number is found in Weyl, who says, " If one wants to speak, all the same, of numbers as concepts or ideal objects, one must at any rate refrain from giving them independent existence ; their being exhausts itself in the functional role which they play and their relations of more or less." [4] If these analyses are to be accepted it would seem that the common aspect according to which the analogous name " number " is to be used to signify the various things will be some kind of relation. The expression " some kind of relation " is here used deliberately, since this " relation " seems also to suffer from ambiguity, systematic or otherwise. If this is true it would seem advisable to obtain some clear notion of what things can be signified by this word " relation " before going on to a further analysis of the word " number."

I. " RELATION " SIGNIFIES THAT ASPECT IN THINGS BY WHICH THEY ARE " TOWARD SOMETHING OTHER "

The word " relation," in its root *refero*, seems to express a kind of " bringing back " or something of the sort. This has led some to the notion that there would be one accident in two different subjects, " one which would stand, so to speak, with one foot in one subject

1. Cf. St. Thomas, *In IV Metaph.*, lect.1, nn.535-536.

2. Ernst Cassirer, *Substance and Function*, Open Court, Chicago, 1923, p.50.

3. Carl B. Boyer, *The History of the Calculus*, Dover, N. Y., 1959, pp.293-4. Italics are mine.

4. Hermann Weyl, *Philosophy of Mathematics and Natural Science*, Princeton, 1949, p.36.

and with the other in the other subject." [1] Used in this way the word " relation " signifies as one thing what is actually two. Thus Weyl says, " Two propositions such as ' 5 follows upon 4 ' and ' 4 precedes 5 ' are expressions of one and the same relation between 4 and 5." [2] Using the word " relation " in this way, Weyl would be led to say that the propositions " John is the father of Edward " and " Edward is the son of John " are expressions of one and the same relation between John and Edward. But this would be to confuse a property of the relative, namely to have a correlative, with the relative itself.

This points to another difficulty in the use of the word " relation " to signify that aspect in things by which they have some reference to another. This reference which a thing has to another is something accidental to the nature of that thing. For example, the fact that John is the father of Edward is something accidental to his nature as a man. The same is true of certain other aspects of John, such as his being white, six feet tall, and the like.[3] Such accidental aspects may be signified either by an adjective or a noun. For example, " white " or " whiteness." The difference here in the mode of signification indicates a very real difference in how the thing is being signified in each case. Thus the word " white " indicates something conceived as existing in some other thing, while the word " whiteness " lacks this notion of existence. This is further shown by the fact that " white " is said to signify concretely, while " whiteness " is said to signify abstractly. The word " relation " is similar. The word " relation " signifies abstractly or without the added reference to something in which it might exist, while the word " relative" signifies concretely or with this added reference.[4] Now since the present analysis is ordered to finding out just what there is *in things* by which they have a common aspect, it would seem that the word " relative " would better direct the attention to this than would the more abstract word " relation." The latter word indicates the manner of conceiving this aspect rather than the aspect itself in things. However, the word " relative " also involves a certain ambiguity in the present instance. It indicates a capacity for the aspect of referring to another rather than the actual reference itself.[5]

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1. LEIBNIZ, in No. 47 of his fifth letter to Clarke, quoted by Weyl, *op. cit.*, p.4.

2. *Ibid.*

3. Weyl and others who would hold that number is relation would probably insist that this is precisely the point at issue : Neither relation nor number are accidents of anything but rather the nature itself. The answer to this must be postponed until it has been determined what is and what is not included in the notion of " relation." For the moment all that is required is the common notion that what is signified by the name "man" does not require that he be white, a father, and the like.

4. Cassirer seems to confuse this "abstraction" with the abstraction proper to mathematics. Cf. *op. cit.*, pp.15 ff.

5. Cf. CAJETAN, *In Praedicamenta Aristotelis*, M. H. Laurent, Rome, 1939, pp.111 ff.

These difficulties about a word to express this aspect in things by which they actually refer to some other thing were apparent to Plato. He coined the expression πρός τι to direct attention more immediately to that particular aspect in things which he wished to signify. This expression was taken over by Aristotle in his *Categories* and came into the Latin in a literal translation as *ad aliquid*. A literal translation to English would be " toward something other." The use of the expression " toward something other " in place of the word " relation " will either clarify or at least make more obvious the confusion which results when Weyl uses " relation " as he did in the passage cited above. If he is now going to say that the two propositions " 5 follows upon 4 " and " 4 precedes 5 " are expressions of one and the same *toward something other* between 4 and 5, it can be said that this is simply false.[1] The expression " toward something other " manifests two very important aspects of the relative. Webster's Dictionary says, " Primarily *toward* denotes the relation of direction or approach without arrival or attainment." But more than this is needed to indicate the particular aspect of things now under consideration. Being "relative " is not like being " white " or " six feet tall " which latter have a certain absolute existence in the subject. Rather its very being consists in a reference to another. This is what is very aptly expressed by saying *toward something other*. Examples of particular words which signify this aspect in things are " double," " half," " master," " slave," " more," " less," and many others. Whenever these words are used they signify someting in the things signified which is *of* or *to* something else.

II. " RELATION " ACCORDING TO ARISTOTLE

When Aristotle treats *ad aliquid* [2] in the *Categories* [3] he first gives as examples " greater " and " double." Then he goes on to say, " There are, moreover, other *ad aliquid* . . ." and here he gives as examples " habit," " disposition," " sensation," " knowledge," and " position." The obvious difference between these and the first two examples, along with the words used by Aristotle to introduce the later examples indicate that those of the second group are not to be found in this category in the same way. Cajetan [4] argues that Aristotle begins

1. Of course, Weyl and others might argue that this is a case of identity in a genus, as isocele and scalene are the same figure, namely triangle. However his use of the expression "*one* and the same" would seem to indicate numeric unity.

2. Since this analysis is concerned with clarifying certain notions about number, no attempt will be made to coin new words to express " relation." Frequently the expression *ad aliquid*, which was common usage for a long time in latin, will be employed. At other times the word "relation" will be employed when the context is such that its use will not generate confusion.

3. Chapter 7, 6 a 37.

4. *Op. cit.*, pp.114 ff.

here by speaking of *ad aliquid* according to the definition of Plato and later [1] corrects this and compares it with his own definition. Babin, in an excellent commentary on the same passage,[2] suggests that Aristotle in the *Categories* is concerned to show the differences between " relatives proper and relatives improper." [3] This distinction is found in St. Thomas [4] and is expressed by the terms *relativa secundum esse*, for relatives proper, and *relativa secundum dici*, for improper relatives. He says,

They are called relatives *secundum esse* when the names are imposed for signifying the relations themselves ; but relatives *secundum dici* when the names are imposed for signifying qualities or something of the sort principally, upon which nevertheless relations follow.[5]

Babin, in his text, then analyses the text of Aristotle on *ad aliquid* in Chapter 15 of Book V of the *Metaphysics*. Here certain things are brought out which are vital to the question of whether relation can be the aspect by which " number " can be used as an analogous name. In this part of the *Metaphysics* Aristotle treats those aspects in things which furnish a foundation for relations. As Babin says,

. . . this text of the *Metaphysics* supplements that of the *Categories* on many important points : in the number and nature of the different species of relatives, on their fundaments and on the nature of the relationship between proper and improper relatives. Thus we learn that there are three kinds of relatives whose distinction rests on the particular nature of their respective

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1. *Categories*, 8 a 12.

2. A. Eugène Babin, *The Theory of Opposition in Aristotle*, Notre Dame, Ind., 1940, pp.1-33.

3. " Thus, this grammatical analysis reveals a clean-cut distinction between the two definitions of relatives given by Aristotle, and between the relatives expressed by the one and the other definition. And since what is essentially such is primary to what is just said to be such, it is convenient to call the relatives of the second definition relatives proper, and and those of the first definition relatives improper. However, such a distinction, although essential, is not one between two species but rather between a genus and a species, so to speak. Indeed, whereas the first definition is true of all relatives, namely, proper as well as improper, the second is true only of the relatives proper, as Aristotle has it at the end of Chapter 7. All relatives, he says, can be said of other things, but not all relatives have their whole being in this relation, just as all men are said to be animals but not all animals are men." *Ibid.*, pp.13-14.

4. *De Pot.*, q.7, a.10, ad 11um.

5. " Dicuntur enim relativa secundum esse, quando nomina sunt imposita ad significandas ipsas relationes : relativa vero secundum dici, quando nomina sunt imposita ad significandas qualitates vel hujusmodi principaliter, ad quae tamen consequuntur relationes." *Ibid.* St. Thomas here gives this as a distinction between two species rather than that of a genus and a species, as indicated by Babin and as it might appear to be in the text of Aristotle in the *Categories*. Cajetan, in his commentary on Aristotle cited above follows the distinction of St. Thomas. Thus it would seem that it would be better to say that " relatives " may be either those *secundum esse* or those *secundum dici*, as indicated by St. Thomas.

fundament : quantity as one and many, quality as active and passive, being as measure and measured.[1]

In discussing the modes of those things which are called *ad aliquid secundum se* Aristotle takes up first those which are found in quantity. He distinguishes those which follow number absolutely from those which follow one absolutely. Those which follow number absolutely may follow either from a comparison of a number to a number or from a comparison of a number to one. Either of these last two comparisons may be determinate or indeterminate, as will be seen later. Those which follow the one absolutely are the comparisons of equal, which is identity in quantity ; like, which is identity in quality ; and same, which is identity in substance. Then he says that " one is the principle and measure of number, so that all these relations imply number, though not in the same way."

The explanation of St. Thomas of this passage [2] is very interesting. He begins by pointing out that all measure found in continuous quantity is in some way derived from number and therefore the relations in continuous quantity are attributed to number. Then he divides the relations found in number. The first division is into the relations of equality and inequality. Those according to inequality are further divided into the exceeding and the exceded, each of which has five species. For the exceeding these will be (according to modern notation) :

1) $\frac{xn}{n}$ $(n=1, 2, 3, \ldots)$

2) $\frac{x+1}{x}$ (x is greater than 1)

(y is greater than 1)

3) $\frac{x+z}{x}$ (z is greater than 1 and less than x or y in the denominator)

4) $\frac{xy+1}{y}$

5) $\frac{xy+z}{y}$

The five species of the exceded are, of course, the reciprocals of these. He says the first of these proportions, is the relation of a number to

1. *Op. cit.*, p.33.

2. *In V Metaph.*, lect.17, nn.1006-1011.

unity, since the exceeding will always be a number (more than 1) and the exceeded with which it is compared will always be a submultiple of it and thus will measure it as 1 measures the other numbers. The other four proportions are all relations of a number to a number rather than of a number to 1. Further, following Aristotle, he says that to state the proportions as was done above is to express the proportion indeterminately. When numbers are substituted in the equations the proportions or relations are then expressed determinately. Then, turning to continuous quantity, he points out that some continuous quantities when compared to each other do not have relation of a number to one or of a number to a number, either determinately or indeterminately. These are the incommensurables, as the relation of the side of a square to its diagonal. However, he says that even though these are not numerical they are, nevertheless, said to be proportionate as the containing to the contained.[1]

This species of relation, founded upon quantity, would seem to offer much as a foundation for the particular kind of relation which could form a basis for the multiple uses of the word " number." However, there are still many difficulties. For example, it is not easy to see how " complex numbers, quaternions and other hypercomplex numbers, modular numbers and transfinite cardinal and ordinal numbers " could all be considered according to the relation of containing to the contained. Another difficulty arises from the fact that Aristotle bases these relations on the one and number as found in quantity. This one and number is, in other places,[2] assigned to the category of quantity which, he says, is something absolute and not relative. Nicomachus, also, in his Arithmetic, speaks of the above-mentioned proportions as " relative number " and contrasts them to that which he calls " absolute number." [3] Thus it would seem that this analysis must be carried further.

The next foundation of relatives is found in things that are active and passive.[4] Such relatives are said in two ways, either according to the active or the passive potency or according to the acts of these potencies. Thus the calefactive is referred to the calefactable as an active to a passive potency, while the heating is referred to the being heated as the acts of these potencies. This foundation for relations would not seem to be too fruitful in the analysis of number, since num-

1. " Quantitas enim qualiscumque accipiatur, vel est aequalis, vel inaequalis. Unde, si non est aequalis, sequitur quod sit inaequalis et continens, etiam si non sit commensurabilis." *Ibid.*, n.1021.

2. *Categories*, 4 b 20 ff. *Metaphysics*, 1020 a 7 ff.

3. NICOMACHUS OF GERASA, *Introduction to Arithmetic*, Bk.II, ch.1. In *Great Books of the Western World*, Vol.II, p.829.

4. ARISTOTLE, *Ibid.*, 1021 a 15.

bers can hardly be considered as acting or being acted upon in any real sense. As Aristotle states it :

Numerical relations are not actualized except in the sense which has been elsewhere stated ; actualizations in the sense of movement they have not.[1]

The third and last mode of those things that are called *ad aliquid secundum se* has as its foundation the measure and the measured. The relatives of this species are quite different from those found in quantity and in action and passion. The double is related to the half and conversely ; so also the father to the son and conversely. However, knowledge and sensation, which are relatives in this third species, are related to the knowable and the sensible ; but the knowable and the sensible are not related to knowledge and sensation (i. e. by any real relation), rather the knowable and sensible are called relative because something is referred to them.[2] The actions of knowing and sensing do not produce a change in the object known or sensed, precisely as they are known or sensed, because these actions remain in the agent and do not affect the object. Along with the knowable and the sensible, Aristotle here mentions the measurable as that which is relative because something else is referred to it. At first glance one might think this was a slip of the pen and suppose that what Aristotle really must have meant to say was that the measure is what is relative because something else is referred to it. The measure by which something is measured is always, as measure, something absolute. If, then, there is to be something relative, it will have to be found in the measured. Thus, as the knowable and the sensible are not changed by the knowing or the sensing, so also the measure is unchanged by the measuring. However, those who " correct " Aristotle so precipitously incur a grave risk of falling into error. In Book *X* of the *Metaphysics*, where he gives a complete explanation of the notion of measure and the measured, he also explains what is being said here in Book *V* :

Knowledge, also, and perception, we call the measure of things for the same reason, because we come to know something by them — while as a matter of fact they are measured rather than measure other things. But it is with us as if someone else measured us and we came to know how

1. *Ibid.*, 1021 a 19. However, it is interesting to notice that most modern mathematicians choose the relations of this foundation, " father and son," " husband and wife," to exemplify what they mean by relation. Are they perhaps moved by some Freudian impulse in this choice to insinuate a mode of " action " and " passion " into an otherwise " abstract " and " cold " science ?

2. Aristotle, *ibid.*, 1021 a 30. Cf. also St. Thomas, *In V. Metaph.*, nn.1026-1029.

big we are by seeing that he applied the cubit-measure to such and such a fraction of us.[1]

Thus it would seem that in Book V Aristotle exemplifies the third species of relatives by the knowable, sensible, and measurable because of the way we speak of knowledge and sense as measuring their objects.

III. THE NOTION OF MEASURE BELONGS PROPERLY TO QUANTITY

It should be noticed that the relatives of the third species are of the kind which Babin called " improper " relatives and St. Thomas and Cajetan spoke of as relations *secundum dici*. Also in the *Categories* Aristotle called these " other " relatives. Unlike the relatives of the first two species, these latter do not have correlatives that are relative in the same way. The reason for this is to be found in the foundation by which these are said to be relative, namely in the measure and the measurable. Thus, while measure is able to found a relation, the measure, as measure, must be something absolute. Aristotle says,

> Measure is that by which quantity is known ; and quantity, as quantity, is known either by a ' one ' or a number, and all number is known by a ' one.' Therefore all quantity, as quantity, is known by the one, and that by which quantities are primarily known is the one itself ; and so the one is the starting-point of number, as number.[2]

There seem to be at least two things in this definition which require some further clarification. The first is that measure is defined according to how something is made known, and the second is the notion of number as a measure. When he speaks of number here, he seems to be speaking of something absolute, since both the " one " and " number " can be measures.

Aristotle comes to the notion of measure here in Book X because he is considering the notion of unity. That which is one seems always to be a kind of measure. The one which is primarily measure is found first in the genus of quantity and from this is applied to the other genera.[3] Thus in continuous quantities we use some small portion of the line, surface, etc., as a unity to measure the whole. From the genus of quantity the notion of measure is transferred to the other genera,

1. *Ibid.*, 1053 a 31. Incidentally, those who would thus correct Aristotle might do well to notice the passage which immediately precedes this quotation, where he very carefully considers whether the measure of units is a unit or whether we should say the measure of number is number.

2. *Ibid.*, 1052 b 20. Notice that he says here, "as quantity" and "as number" to indicate that there is another way in which quantity and number can be known, namely by their properties and accidents. Cf. ST. THOMAS, *In X Metaph.*, lect.2, n.1938.

3. Cf. *Ibid.*, 1052 b 25 ff.

for example quality. We can take one color, or one intensity of a a color, and use this to " measure " other colors or intensities. When we use the word " measure " to refer to the measurement of quantity by the one or number, and again to refer to the measurement of qualities by a color, the word " measure " becomes an analogous name. This is what happens when the word " measure " is applied to knowledge and the knowable. Since the definition of measure is " that by which quantity is made known " there are two elements in the definition. The first is the notion of quantity, which is the proper genus of measure, the second is the notion of something becoming known. Now quantity is variously defined as " parts outside of parts " and " that which is divisible into those things which are in it," but quantity, precisely as an accident, is the measure of substance.[1] It is for this reason that quantity is the proper genus of measure. Thus it is that measure is found in those things which are actually quantified and among these, primarily in discrete quantity or number, and it will be the one, principle of number, which is the primary measure.

On the other hand, since the notion of measure also includes the notion of making something known, there are two ways in which one thing can measure another. The first is in quantity where the unit or measure is compared to the indeterminate quantity and, by being taken so many times, finally equals it. In another way one thing is said to measure another when the first is the reason for knowing another. This happens when we are led to the knowledge of one thing by some other thing. In this way one thing is said to measure another, not absolutely, but by means of knowledge. It is in this way that knowledge and sensation are said to measure the knowable and the sensible

Now when quantity is spoken of as the measure of substance, the substance referred to is, of course, material substance. If there are substances which are not material they will not be measured quantitatively. It is only by means of quantity that many individuals of the same species can be distinguished.[2] Substances which were not material would each constitue a separate species and there could not be a plurality of individuals of the same species.[3] Moreover, when we speak of distinction in some multiplicity, this can refer either to that found in the things themselves or that by which these things are known. Quantity, as the measure of substance, distinguishes the multiplicity of individuals in the same species (having the same

1. *In I Sent.*, D.19, q.4, a.2, c. Cf. also *Ia Pars*, q.28, a.2 ; JOHN of S. THOMAS, *Curs. Theol. II*, disp.9, a.1, Solesmes, p.56 ; and especially, *In IX Metaph.*, lect.1, n.1768.

2. Cf. Charles DE KONINCK, "Abstraction from Matter", *Laval théologique et philosophique*, 1957, vol.XIII, n.2, pp.159 ff.

3. Cf. ST. THOMAS, *De Spirit. Creat.*, a.7.

form) as that multiplicity exists in the things themselves. It is by reason of his quantity that Plato is distinct from Socrates. The fact that one or the other, or both, may be increasing or decreasing his quantity at the time does not affect the fact that their quantity is that by which they are distinguished.[1] Therefore when it is said that measure is that by which quantity is known, it will be necessary to distinguish between that quantity by which the things themselves are distinguished and that same quantity as known. When three men or three horses are known the unit or measure by which they *are* three and that by which they *are known* to be three coincide. On the other hand, when it is said that Socrates is six feet tall or that a piece of cloth is three yards long, no such distinction in the things themselves is found. In the first case there is an actual quantitative multitude of three men or three horses, while in the " six feet " of Socrates or the " three yards " of cloth the quantitative multitude is only potential. It is made actual only by the act of knowing or measuring. Yet in each case there is a number that is the measure of the multiplicity, whether that multiplicity is actual or potential. This number, which is number applied to things, is called by St. Thomas *numerus numeratus* as opposed to *numerus quo numeramus*.[2]

IV. ABSOLUTE NUMBER IS THE SUBJECT OF THE SCIENCE OF ARITHMETIC

That which is measured or numbered, the number of things or numbered number, may be either continuous or discrete. Thus when a line, surface, motion or the like is measured, that which is numbered is continuous ; but when a group of men, horses, trees or the like is numbered, that which is numbered is discrete. On the other hand, *numerus quo numeramus* is always discrete.

For Aristotle and St. Thomas, number arises from the division of the continuum.[3] However, there is a difference in the way number or

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1. Cf. DE KONINCK, *op. cit.*, p.160.

2. *In IV Phys.*, lect.17, n.11. St. Thomas here follows the distinction made by Aristotle (219 b 5). In other places St. Thomas calls this " *numerus quo numeramus* " either *numerus absolutus*, *numerus simplex*, *numerus simpliciter*, *numerus in intellectu*, or *subjectum Arithmeticae*. *Vd.*, De Pot., q.9, a.5, ad 6um et 8um ; *In I Sent.*, D.24, q.1, a.2 ; *Ibid.*, D.19, q.2, a.1, and q.4, a.2 ; *Ia Pars*, q.30, a.1, ad 4um ; *Q. Quodl. X*, q.1, a.1 ; *In X Metaph.*, lect.4, nn.1993 ff. It seems possible that this " absolute number " might be what modern mathematicians refer to as " the positive integers " or " natural numbers."

3. ARISTOTLE, *III Phys.*, 207 b 10 ; ST. THOMAS, *In III Phys.*, lect.12, n.5 ; *Ia Pars*, q.30, a.3, c.; *In I Sent.*, D.24, q.1, a.2, ad 2um ; *In II Sent.*, D.3, q.1, a.3, ad 1um ; *De Pot.*, q.9, a.5, ad 8um. In what follows, *numerus quo numeramus* will be referred to as " absolute number " to avoid confusion. By this will be meant discrete quantity abstracted from things, the subject of the science of Arithmetic.

quantitative multiplicity is produced in itself and the way we come to know it, as was said above. St. Thomas explains the genesis of the idea of number, which is a species of quantity, as follows : The intellect arrives at the notion of one before that of multitude, although in sense and imagination it is the converse. The intellect first understands being and non-being and this is sufficient for the notion of unity, such that it understands being to be one. From this the intellect is able to define multitude as that made up of units one of which is not the other. Then, since this being, and beings, is *ens concretum in quidditate sensibili*, the notion of multitude thus arrived at is a quantitative multitude. The notion of number follows upon the notion of the division of this multitude when the multitude is considered as measured by the unit.[1] Thus while number is *caused* by the division of the continuum, still we come to know discrete quantity before the continuous in the sense that we can measure and thus number the continuous. This discrete quantity which is first known is not absolute number, the subject of Arithmetic. The latter is arrived at only when we abstract from the existence of number in things and consider it *simpliciter*. It is this absolute number which Nicomachus speaks of in his *Arithmetic* [2] and he begins the science by discussing its properties, as odd and even, prime and composite, etc. Among modern mathematicians this part of mathematics is known as Diophantine analysis insofar as it limits the solutions of equations to positive numbers or integers. Analysis in absolute number considers the absolute one, principle of number, as the measure of number. Furthermore, the measure must always be homogeneous with the measured, which leads Aristotle to say that the measure of units is a unit. As he says,

We must state the matter so, and not say that the measure of numbers is a number ; we ought indeed to say this if we were to use the corresponding form of words, but the claim does not really correspond — it is as if one

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1. *In I Sent.*, D.24, q.1, a.3, ad 2um. "... Primum enim quod cadit in apprehensione intellectus, est ens et non ens : et ista sufficiunt ad definitionem unius, secundum quod intelligimus unum esse ens, in quo non est distinctio per ens et non ens, et haec scilicet distincta per ens et non ens, non habent rationem multitudinis, nisi postquam intellectus utrique attribuit intentionem unitatis, et tunc definit multitudinem id quod est ex unis, quorum unum non est alterum ..." *Ibid.*, ad 3um. "... Est enim duplex divisio : scilicet secundum quantitatem ; et talis divisio consequitur rationem multitudinis, eo quod rationem multitudinis communiter acceptae sequitur ratio numeri, prout est species quantitatis, secundum quod addit rationem mensurae ; unde dicit Philosophus, *X Metaph.*, quod numerus est multitudo mensurata per unum ; et rationem numeri sequitur intellectus divisionis continui. Ratio enim divisionis, et quantitatis, et mensurae, secundum Commentatorem, *X Metaph.*, prius invenitur in quantitate discreta quam in quantitate continua ..."

2. *Op. cit.*, pp.821, 829. In Book II, ch.22, he gives priority to the arithmetic proportion because "Nature shows it forth before the rest" and " in the natural series of simple numbers, beginning with one, with no term passed over or omitted, the definition of this proportion alone is preserved ..."

claimed that the measure of units is units, and not a unit ; number is a plurality of units.[1]

So far in this analysis it has been assumed that the one, principle of number, is distinct from the one, convertible with being. However, in the history of thought this distinction has not always been too clearly evident. St. Thomas speaks of this confusion as it existed in Plato and the Pythagoreans, on the one hand, and in Avicenna, on the other.[2] Plato and the Pythagoreans had the notion that the only " one " was that which is convertible with being, while Avicenna held that every " one " is the same as the one, principle of number. Father Weisheipl clearly shows that, in spite of the work of St. Thomas and St. Albert, this confusion runs through the thinking of the Middle Ages.[3] In addition, Burtt, in his classic work on modern science, shows that this confusion is at the basis of much thinking about mathematics, metaphysics and natural science to the present time.[4] In view of these confusions it might be well to consider a text of St. Thomas where he clearly distinguishes these notions.

. . . One is said in two ways, namely that which is the principle of number, and that which is convertible with being. Speaking of the one which is the principle of number, as was said, this implies something added over and above being which is called one, namely the notion of measure. Thus this one can be considered in two ways : either according to that which it is or according to that which follows the understanding of it, namely a kind of relation. If (it is considered) in the second way (the one) is opposed to the numeric multitude relatively, as principle to the principiated, as point to line, part to whole, and more properly measure to measured. If (it is considered) in the first way, this also is twofold : because either the one itself will be considered with precision, namely that it is unity only, and thus it will have a disparate opposition of measure with respect to the other numbers ; because any number according to the quiddity of its species has a special reason of measure, just as opposed species are disparate, and such opposition is reduced to contrariety as a principle, because disparate species are distinguished by the different contraries by which the genus is primarily divided, as is proved in *X Metaph.* Or (the one itself will be considered) without precision, and in this way unity is not opposed to number, but rather is something constituting it. However, if we speak of the

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1. *Metaph.*, 1053 a 27.

2. *Ia Pars*, q.11, a.1, ad 1um ; *In IV Metaph.*, lect.2 ; *De Pot.*, q.9, a. 7, c.

3. James A. Weisheipl, o.p., *Physical Theory in the Middle Ages*, Sheed and Ward, N. Y., 1960, pp.50ff. *Vd.* also his article, " Albertus Magnus and the Oxford Platonists," *Proc. Am. Cath. Phil. Assn.*, vol.32, 1958, pp.124-139.

4. E. A. Burtt, *Metaphysical Foundations of Modern Science*, Doubleday, N. Y., 1954. His whole text is illuminating when considered in the light of this distinction, but expecially what he has to say about Kepler, p.67. It would seem that it might be possible to work out a very coherent History of Philosophy using the confusions in the various thinkers in their notion of the "one" as a basis of comparison.

one which is convertible with being, then the one has the nature of privation, as was said, with respect to the division which is preserved in the multitude. In this way it is opposed to the multitude as privation to habitus, as the Philosopher says in *X Metaph.* In this way also the equal is opposed to the great and small as privation. Nor is the one the privation of that multitude which it constitutes, but rather of the multitude which is denied to exist in that which is called one. For the one is not deprived of all division by its very nature, rather it suffices for its nature that whatsoever division (pertaining to it) be removed. It is in this way that this one can be a part of a multitude, and the multitude itself is said in a certain way to be one, namely insofar as it is something not divided, as least according to the way the intellect considers it as an aggregate.[1]

Thus there is a clear distinction between these two kinds of unity. The one, principle of number, is a measure while the one convertible with being is the negation of division.

V. NUMERUS NUMERANS IS SOMETIMES CONFUSED WITH NUMERUS ABSOLUTUS

John of St. Thomas, at the beginning of the 17th Century, speaks of a " common distinction between numbered number and numbering number " that existed at the time.[2] It would seem that this " numbering number " is what Aristotle and St. Thomas referred to as absolute number, *id quo numeramus.* In the text just cited he says

1. *In I Sent.,* D.24, q.1, a.3, ad 4um. " Unum dupliciter dicitur, scilicet quod est principium numeri, et quod convertitur cum ente. Loquendo de uno quod est principium numeri, ut dictum est, ponit aliquid additum supra ens quod dicitur unum, scilicet rationem mensurae ; unde hoc unum potest dupliciter considerare : aut secundum id quod est ; aut secundum id quod consequitur ad intellectum ejus, scilicet relationem quamdam. Si secundo modo, sic opponitur multitudini numerali relative, sicut principium ad principiatum, ut punctus ad lineam, et sicut pars ad totum et magis proprie sicut mensura ad mensuratum. Si primo modo, tunc dupliciter : quia vel considerabitur ipsum unum cum praecisione, scilicet quod est tantum unitas, et sic habebit disparatam oppositionem mensurae ad alios numeros ; quilibet enim numerus, secundum quidditatem suae speciei, habet specialem rationem mensurae, sicut species oppositae sunt disparatae, et talis oppositio reducitur ad contrarietatem, sicut principium : quia species disparatae distinguuntur differentiis contrariis, quibus primo dividitur genus, ut probatur *X Metaph.*, text 24. Vel sine praecisione, et sic unitas nullam oppositionem habet ad numerum, sed est constituens ipsum. Si autem loquimur de uno quod convertitur cum ente, sic unum habet rationem privationis, ut dictum est, respectu divisionis quae salvatur in multitudine ; et sic opponitur multitudini, sicut privatio habitui, ut dicit Philosophus, *X Metaph.*, text 9. Unde etiam aequale opponitur magno et parvo, sicut privatio. Nec unum est privatio illius multitudinis quam constituit, sed multitudinis quae negatur esse in ipso quod dicitur unum. Non enim de ratione sua unum privat omnem divisionem ; sed sufficit ad rationem ejus, quaecumque divisio removeatur. Et inde protest esse quod unum est pars multitudinis, et quod ipsa multitudo dicitur quodammodo unum, prout scilicet aliquid non dividitur, ad minus secundum intellectum aggregantem . . ."

2. *Curs. Phil.*, t.I, (ed. Reiser), p.552.

numbering number is the "*ratio numerandi in intellectu, ut duo, tria, quatuor, etc.*" However, a few lines later he says "... *aliquando ipsa discretio quantitatis respective ad substantiam, quae illi subicitur, dicitur numerus numerans.*" This last would seem to be quantity as a measure of substance and thus something in the things themselves. If numbering number is to be applied to both of these the term will be equivocal. Then in his *Cursus Theologicus* he has another very interesting observation. He says,

> If we speak of numbering number abstractly, as one, ten, one hundred, one thousand, etc., this mensuration does not seem to belong only to the quantitative multitude or number, but also to the transcendental multitude. Thus also it is applied to the angels ; but yet it is conceived by us after the manner of quantitative multitude and number by reason of the imperfection of our concepts.[1]

This last seems to be based on what St. Thomas calls "*numerus qui est in ratione tantum* which he opposes to *numerus simpliciter*.[2] This notion of number which can be applied to a transcendental multitude, or number which is "*in ratione tantum*" seems to have had the greatest appeal for modern philosophers of mathematics. It has been pointed out by De Koninck [3] that modern mathematicians are for the most part concerned with what the Greeks called *logismos*. This art, as he says, abstracts from the distinction between *per se* and *per accidens*, either as to being or as to unity. However, the notion of measure here is quite tenuous.

From what has been said it might seem that one kind of relation that might serve to unite the various significations of the word "number" could be that of measure to measured. However, besides the fact that the word "measure" so used would itself be an analogical or even purely equivocal name, since it is applied to things in different genera and even to things which cannot be in a genus, there is a difficulty about the numbering number mentioned above. If the notion of numbering number is to be that which is applied to a transcendental multitude, the very basis of measure would seem to be absent. Measure requires some kind of homogeneity. In the homogeneous multitude each part has the same form and thus any part can measure the whole. In a transcendental multitude each part has a different form and none of the forms can measure the others in any proper way. If these latter should be considered as in some way measured with a numbering number which is *in ratione tantum*, such mensuration would

1. T.II, (ed. Solesmes), p.103.

2. *In I Sent.*, D.24, q.1, a.2. "... Numerus divinarum personarum est medius inter numerum qui est numerus simpliciter, et numerum qui est in ratione tantum ..."

3. Charles DE KONINCK, "Random Reflections on Science and Calculation," *Laval théologique et philosophique*, 1956, vol.XII, n.1.

be purely extrinsic to the things of the multitudes and have as its basis only the act of the mind making the relation.[1] Added to these difficulties is the fact that some " numbers " combine variously elements of absolute number, relative number, numbered number, along with some aspects of numbering number.[2] For any complete analysis of the word " number " it would be necessary to investigate each of its various forms as used in modern mathematics.

However, although the present analysis cannot pretend to any such perfection, certain things have, it seems, been brought out. The term " numbering number " has been shown to apply both to absolute number and " transcendental " number, as well as having a meaning which would be that of quantity taken as the measure of substance. Absolute number has been shown to be the subject of the science of Arithmetic and to have certain properties belonging to it absolutely, as well as certain other properties which arise by the comparison of a number to a number or of a number to 1. Further, absolute numbers may be applied to physical things as measure and thus give numbered number, and this either as physical things are discrete or continuous. If many difficulties remain, and they do, we can only repeat the observation made by Aristotle as he ended his consideration of *ad aliquid* in the *Categories* : " It is perhaps a difficult matter, in such cases, to make a positive statement without more exhaustive examination, but to have raised questions with regard to details is not without advantage." [3]

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1. In a sense this is an even more tenuous *relatio rationis* than that which relates a thing to itself as the " same," as St. Thomas says, "... *Sicut aliquis est idem sibi realiter, et non solum secundum rationem, licet relatio sit secundum rationem tantum ...*" *De. Pot.*, q.8, a.11, ad 3um.

2. One example of this might be the calculus in a strict sense, which combines measures of motion and distance (numbered number) with a consideration of proportion (relative number) along with " velocity in an instant " (*in ratione tantum*).

3. *Ibid.*, 8 b 20.