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#### Article abstract

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## A Network Flow Model for Irrigation Water Management

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#### **Abstract**

Irrigation water management plays a crucial role in the growth and prosperity of countries like India. Optimization Techniques can be effectively used in the management of irrigation water. Motivated by a real crisis in Andhra Pradesh, India, the authors made an attempt to provide scientific solution to the problem of management of Pennar Delta System of Nellore District in Andhra Pradesh. The problem concerns the management of water distribution and scheduling for given requirements and availabilities of water at various nodes of the irrigation network of the system. This article provides a model and framework for the problem in question. The problem is formulated as a dynamic minimum cost network flow problem and provides an approach to solve the problem using static network flow models. A need based software is also developed to solve the network flow problems. Some issues in the programming are discussed.

Key words: Irrigation water management, network flow models, and software solutions.

#### 1. Introduction

Efficient management of water for irrigation plays crucial role in countries which are facing severe water crisis. Several authors have studied the need and importance of water management for irrigation purposes ([3], [7], [14]). The motivation for the work in this article has stemmed from a serious water crisis that the state of Andhra Pradesh in India was facing. This article proposes a network based optimization model framework for management of irrigation water. The irrigation water management requires good statistical data maintenance systems and information technology for effective decision making and implementation. Prior to early 1990s the information technology was in the developing stages and this has partly been responsible for the lack of efficient statistical systems. One of the other main causes for this lacuna is the lack of awareness (among the administrators) of scientific methods and their necessity in decision making processes and management. But, with the level of advancement and growth of information technology that we have today, we should quickly design, develop and establish efficient statistical systems and implement them using electronic media so that our

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resources are best utilized. This can be achieved only if the people in the scientific and governing communities come together and put conscious and untiring efforts to bridge the gaps.

The prosperity of a country like India where agricultural produce is about 14% of its GDP depends largely on efficient management of water resources and proper irrigation methods. Therefore, optimum utilization of resources pertinent to agricultural sector is very important for the growth. A large part of India's agricultural fields are fed by irrigation (http://planningcommission.nic. in/plans/planrel/fiveyr/11th/11\_v3/ 11\_v3\_ch2.pdf). Water supplied from rivers and reservoirs to irrigated lands is undoubtedly the most important of agricultural resources. Due to ecological disturbances, the availability of water is becoming increasingly more difficult year after year. Therefore, it is utmost important for us to manage irrigation water in the best possible way ([6]). This becomes particularly more relevant in the present days global scenario of depleting water resources and acute water scarcity problem ([14]).

Pennar Delta ayacut (irrigation lands) of Nellore district in Andhra Pradesh serves about 2,50,000 acres of agricultural lands. Due to severe summer and draught

conditions, this ayacut had faced acute water shortage problem during the past and had led to agitations by the farmers and the district administration was under tremendous pressure to resolve the imbroglio. Added to the acute water shortage problem, lack of adequate information on actual amount of water available and the water requirements (determined by various crop patterns and their extents) had complicated the problem. Motivated by this imbroglio, we conceived the idea of initiating a project towards developing a scientific approach to address the problem. When the idea was proposed to the concerned authorities in the district administration, they readily came forward and welcomed the initiative. Consequently this project has been initiated taking the Pennar Delta ayacut as the source for model building. Typically a project of this nature has to address a number of issues such as development of systems and procedures for acquiring and monitoring necessary data and information on electronic media, making the same available to all concerned personnel for planning and monitoring, developing efficient optimization tools for managing the resources, training the personnel in the administration, educating the farmers, advising and guiding them about crop patterns and scheduling etc. A complete solution to this problem requires concentrated efforts by a team of specialists from different areas and other resources such as financial support from the Government, etc. Optimization techniques have been used to model the irrigation water utilization ([8], [9], [12], [15]). We have initiated the project with a limited scope so as to create awareness among the administrators and induce them into taking up the project in its entirety. The scope of this article is limited to developing an optimization model that will generate a water distribution plan for the ayacut. An attempt is also made to develop a software to aid the management. The details are elaborated in the next section.

### 2. The Irrigation System

The model irrigation system considered for this work has been derived from the Pennar Delta System (PDS). A major or medium irrigation system such as this consists of a main reservoir, main channels, branch channels, distributary channels and several storage and/or regulatory points. Water is carried to the agricultural fields (AFs) through these channels. A pictorial description of the system is shown in Figure 1.

The collection of all AFs is called the project ayacut. Each channel is designed for a specific capacity to

meet its intended requirements. The channel parameters such as cross sectional area, slope etc., will determine the delivery rates (usually measured in cubic feet per second (cusecs)). The release of water in any channel is controlled by the regulators by opening their gates partially or fully. Generally the regulators have storage capacities. A number of factors such as crop pattern variation, timing of crops, soil conditions (water absorption properties), etc., determine the water requirements of the agricultural fields. The normal practice of supplying water to AFs is to declare the opening and closing dates of various regulators. This results in uniform supply of water to AFs which may be detrimental to the crops. This is because the water requirements of crops vary with different growth stages of the crops.

An important feature of the irrigation system under consideration is that every AF is connected to one and only one channel in the network. Technically speaking, the irrigation network is a tree in Graph Theory terminology ([16]).

One of the constraints in releasing water at a regulator into a channel is that there should be certain minimum quantity of water at the regulator so that the required water head is there for the water to flow into the channel. For example, at Nellore regulator there are two branch channels. One of these two channels, the Kanupur canal, is at a higher altitude than the other. In order to supply water in this channel, one has to build up the water head above the channel bed level at the regulator and then release the water.

The calculated quantity of water supplied in a channel and the actual quantity of water supplied are usually at variance. There are several reasons for this. Though the channels are designed for predetermined capacities, the actual capacities may be at variance with designed capacities. Silt formation in the channels, lack of proper maintenance, etc., also affect the water quantities supplied. In addition, water losses take place both in transmission as well as at storage points.

### 3. Objectives and Scope of the Project

As briefed in the introduction, a complete solution to the irrigation system involves a number of issues such as development of systems and procedures for acquiring and monitoring necessary data and information on electronic media, making the same available to all concerned personnel for planning and monitoring, developing efficient optimization tools for managing the resources, training the personnel in the administration, ed-

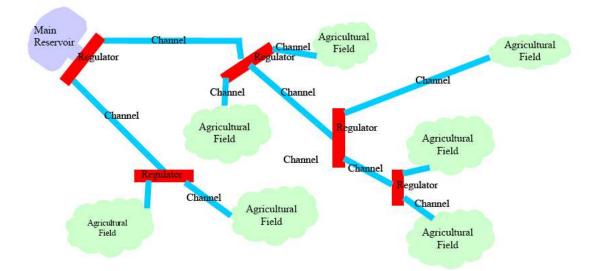


Fig. 1. Pictorial Description of Reservoir Irrigation System

ucating farmers, advising and guiding them about crop patterns and scheduling etc. Since such a task is well beyond the scope of present study, initially this project was taken up with the following limited objectives:

- Develop an optimization model for the following problem: Given the water requirements of all AFs over a specified period, the quantities of water available at the main reservoir and other storage points, and the basic network of the irrigation system, build an optimization model to determine optimal utilization of water and the complete time schedule for the optimum distribution of water.
- To develop software to solve the problem with given input data so that it can be used at least for the planning purposes at the macro level or carrying out the sensitivity analysis.

Keeping the above objectives in mind, steps were initiated to collect the basic information from PDS on the following items:

- Reservoirs: locations, capacities (designed and actual), storages.
- Canals (major, minor and sub minor): start and end points, flow rates (designed and actual), officials (posts) in-charge.
- Crop (final) units: survey number, area, types of crops, water requirements, wetting times, crop yields (expected and past data), water source (which regulator / canal).
- Control structures: regulators, sluices.

It has been noticed that the above information is not available in one place and compilation of the same is a very time consuming exercise. The work is still under progress. Pending this, the framework for this problem is developed. This article reports the model development and the development of the software.

#### 4. Formulation and Solution to the Problem

In this section we shall give a detailed description of the irrigation water management problem and formulate the same as an optimization problem. The problem will be described through examples. First let us understand the basic network of the irrigation system. This is shown in Figure 1. For ease of description, the same is reproduced in Figure 2 with regulators and AFs labeled from 2 to 13. The main reservoir is labeled as 1. We shall refer the regulators and the AFs as nodes of the network. Thus, every node is identified with its label. With this labeling, every channel in the network can be identified uniquely by its tail and head nodes. For example, the channel connecting the regulator 2 and regulator 4 is uniquely identified by the pair (2, 4). Here, 2 is the tail node and 4 is the head node of the channel. Similarly, channel connecting regulator 5 and AF 11 is uniquely identified by the pair (5, 11) with 5 and 11 as its tail and head nodes respectively.

In Figure 2, note that every channel is assigned a pair of numbers. For example, channel (2, 4) is assigned

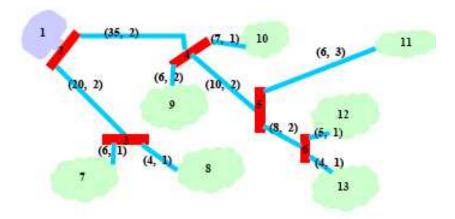


Fig. 2. Basic Network of Irrigation System

the pair (35, 2). Here, the first coordinate, 35, stands for the maximum delivery capacity of the channel per unit time. The second coordinate, 2, stands for the time taken for delivering the water from the tail node to the head node. If we take day as the unit of time, then (35, 2) means that the channel (2, 4) can carry at most 35 units of water per day and it will take two days for the water released at regulator 2 to reach regulator 4.

Generally, regulators have the capacity to store certain amount of water. Therefore, we define  $S_c(k)$  as the maximum amount of water that can be stored at regulator k. Also, when water is stored in reservoirs or at regulators, losses take place due to evaporation. In practice, the evaporation losses are affected by atmospheric conditions and surface area of the water exposed. For the purpose of this project, we shall make the simplifying assumption that the evaporation losses are directly proportional to the quantity of water stored and the duration of the storage. We shall use the notation  $L_c(k)$  to denote the rate of evaporation loss at node k. Water losses take place during transmission as well. We shall use the notation  $L_c(i, j)$  for rate of transmission loss in the channel (i, j).

One of the water management problems here at a very macro level is to determine how much water to supply to each of the AFs over a given period of time (period being a season or a year). The inputs for such a problem will be the total amount of water available at various storage points during the period and the total requirement of water of the AFs. These two issues together with total channel capacities act as the constraints of the problem. The objectives may be defined by the water losses or meeting certain priorities among the AFs. It should be noted that the time schedule does not come into the pic-

ture of this problem. This problem can be formulated as a minimum cost network flow problem and solved. Since time factor is not involved in this problem, these are known as the static network flow problems. However, if the problem involves scheduling activities over a time horizon, one has to deal with dynamic network flow problems. In this project, we are concerned with dynamic network flow problems. A brief description of the static and dynamic network flow problems is given in the next section. See [11] for more details on these problems.

We shall now examine the more important problem of drawing a time-dependent schedule for a given set of requirements and availabilities of the problem. As this problem becomes complex even with a small number of nodes, we shall use a reduced size of the problem shown in figures 1 and 2. The basic network for reduced problem is shown in Figure 3. The number of AFs is reduced to 4 and the nodes are relabeled.

Suppose it is required to plan and develop the water distribution schedule for the month of March in a year. Table-1 below summarizes the water requirements at the AFs and the water availabilities at regulation points. We see that AF5 requires 8 units of water on March 27th and 9 units on March 28th. Similarly, it can be seen from the table that no water is available prior to March 20th but on 20th, 20 units of water is available at regulator 2. On March 22nd there is an additional 55 units of water available at the main reservoir (node 1). Thus, the total water available during March is only 75 units. The total requirement in the month is equal to 72 units.

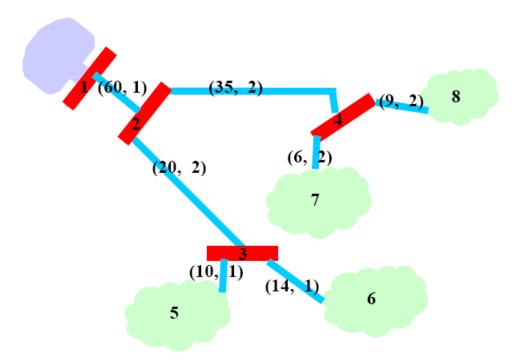


Fig. 3. Basic Network of the Reduced Problem

Table-1: Water Availability and Requirement Data

Tueste 11 Hauter 11 Hauter 11 June 11 June 11 June 1					
Date	Node	Requirement/			
		Availability			
March 20	2 (Regulator)	20			
March 22	1 (Main reservoir)	55			
March 26	AF7	6			
March 27	AF5	8			
March 27	AF6	14			
March 27	AF8	6			
March 28	AF5	9			
March 28	AF6	14			
March 28	AF7	6			
March 28	AF8	7			

Besides the availabilities and requirements, the water distribution schedule has to satisfy the capacity constraints (of storage as well as channel transmission capacities). Table-2 presents the data on storage capacities at the regulator points and also the evaporation losses. As explained, we shall assume that the storage losses are proportional to the quantities being stored. Therefore, we shall represent the storage losses in terms of percentages.

Table-2: Storage Losses in Percentages

Node	1	2	3	4
Storage Loss $(S_c)$	200	20	10	9
Storage Loss $(L_c)$	3	2	1	1

Finally, we need to specify the transmission losses Lc(i, j) of the channels in order to formulate the problem. The transmission losses are given in Table-3 below.

Table-3: Transmission Losses in Percentages

Channel	Transmission Loss	
(1, 2)	2	
(2, 3)	1	
(2, 4)	1	
(3, 5)	3	
(3, 6)	1	
(4, 7)	1	
(4, 8)	2	
-		

The water distribution and supply scheduling problem has to specify the amount of water to be released in each channel on each of the days during the planning horizon. This problem is solved by creating the extended network from the basic network and the input

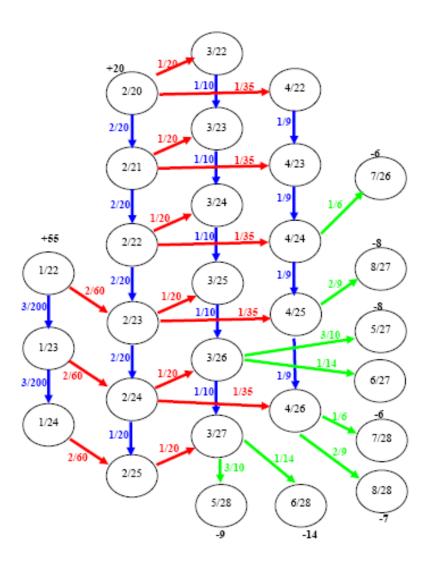


Fig. 4. Extended Network for the Example

data given in Table-1. The extended network is shown in Figure 4. It can be seen that the nodes in the extended network are represented by the node numbers in the basic network coupled with dates. Since there is water available at node 2 on 20th, we create the node represented by 2/20. Since water can reach nodes 3 and 4 on 22nd from 2, the nodes 3/22 and 4/22 are created. Similarly, the other nodes are created as shown in Figure 4. In the water scheduling problem, we should have the option to bring the water to a particular node ahead of time and store it there for later use. As an example, note that nodes 5 and 6 put together require 22 (=8+14) units of water on 27th. Since the transit times to 5 and 6 from 3 are one day each, we must have 22 units of

water on 26th at node 3. As the channel capacity of (2, 3) is only 20 units, we cannot serve 5 and 6 on 27th unless water is brought to 3 (at least 2 units) before 26th and stored there. Suppose we bring some water, say 5 units, to 3 on 25th itself and store it there for a day. In the formulation of the problem we handle this situation by introducing a fictitious channel between the nodes 3/25 and 3/26 (in the extended network) and fix the flow in this channel as 5 units. Since the storage capacity of node 3 is 10 units, we impose the condition that the transmission capacity of the fictitious channel (3/25, 3/26) as 10. All storage decisions are thus represented by the blue colored arcs (arrows) in Figure 4. The red colored arcs represent the usual flow channels between

two regulator points. The flow channels from regulators to AFs are shown by the green color arcs. This way we can understand the extended network shown in Figure 4. Note that two numbers a/b are specified for each of the arcs. Here a stands for water loss and b stands for the storage capacity or the maximum flow capacity of the corresponding arc depending upon whether the arc is a storage arc or a transmission arc. Since storage capacity at regulator 1 is 200 and the storage loss is 3, 3/200 is by the side of the blue arc connecting 1/22 and 1/23, and the arc connecting 1/23 and 1/24. Similarly, since the maximum flow capacity of the channel (2, 3) is 20 and the corresponding transmission loss is 1, we find 1/20 by the side of the arcs which corresponds to (2, 3). Finally, the water availabilities, 55 units and 20 units, are shown at the appropriate nodes in the extended network with a + sign (+ standing for availability); and the AFs requirements are shown with minus signs (- to indicate that these are requirements).

The dynamic network flow problems are typically solved by converting them into static network flow problems. This is exactly what we have done for our problem. The extended network in Figure 4 is actually a dynamic network flow problem presented in the form of a static network flow problem. All the inputs needed for solving the water scheduling problem are shown in the extended network. Once this is ready, we can solve the problem as minimum-cost network flow problem. We take the total evaporation losses as the objective function for this problem. This problem (given in the example) is solved using the computer program developed by us as part of this project. The solution is pictorially shown in Figure 5. In the figure, the blue arcs stand for storage and the green ones for the transmission across channels. The total evaporation losses are 5.35 units under the optimum solution.

So far we have seen two problems in the water management: (i) the macro level planning and (ii) the water distribution and scheduling problem over a time horizon. We shall examine some practical constraints in the latter problem and how to handling these while solving the same.

## 4.1. The Minimum Flow Constraints

In the example we have considered above, we assumed that it is possible to release any amount of water in a channel not exceeding the maximum transmission capacity of that channel. But due to transmission losses, it may be impractical to release small quantities of wa-

ter in the channel. Therefore, just as the channels have maximum transmission capacities; it is natural to consider minimum capacities for the channels. For example, we may insist that the flow in channel (2, 3) should be at least 3 units whenever water is released in this channel. This sort of constraints can be easily incorporated into our formulation because minimum and maximum flow capacities are a part and parcel of minimum cost network flow problems.

#### 4.2. The Level Constraint

The distribution channels starting from a regulator may not be at the same altitude. Recall the Kanupur canal situation discussed in Section 2. Thus, different channels starting from a regulator may need different water heads for getting water. This situation can be handled by introducing artificial arcs in the network with certain minimum flow requirements. We shall illustrate this with an example. Let us suppose that channel (4, 6) in Figure 3 is at a higher altitude and requires additional water head at regulator 4. Since additional head means additional quantity of water, whenever there is water requirement at 8, some minimum level should be maintained at 4. Let us assume that this minimum quantity is 5 units. Since AF 8 requires 8 and 7 units of water on 27th and 28th respectively, we should have 13 (=8+5) and 12 (=7+5) units of water at 4 on 25th and 26th respectively (note that the transit time is 2 days between 4 and 8). We employ the following trick to handle this situation. Introduce a dummy arc in the extended network (in Figure 4) between the nodes 8/27 and 4/26 with a minimum flow requirement of 5 units. Introduce a dummy node 4/27 and declare 5 units of water as a requirement of this node. Connect the nodes 8/28 and 4/27 with another dummy arc and fix its minimum flow as 5 units. This augmented network will take care of the level constraint.

## 4.3. The Group Constraint

In the example, we have specified the AFs requirements day-wise. Instead we may wish to specify requirements of one or more AFs grouped over days. Suppose we want to specify the requirement of AF 5 as 17 units over the two days, 27th and 28th, instead of specifying 8 units on 27th and 9 units on 28th separately. We can handle such constraints by adding a new node with a requirement of 17 units and connect it to the nodes 5/27 and 5/28 using two different arcs. No requirements are

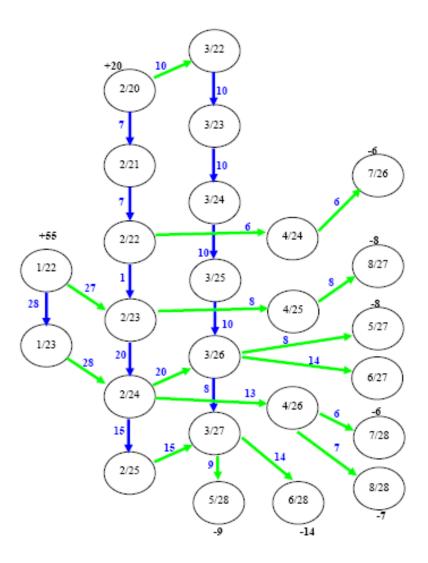


Fig. 5. Solution to the Example

specified at 5/27 and 5/28.

### 4.4. Handling Priorities

In practice, often there is not sufficient water for serving all AFs. In such situations, it is not possible to serve water to all AFs and one has to resort to some sort of priorities based on some evaluation. Accordingly, one can attach some cost to each of the arcs ending with AFs and solve the problem.

In this section we have seen how the water distribution and the scheduling problem can be formulated as a network flow problem and how various practical constraints can be handled. Also, we have seen how a small size problem with only 8 nodes and 7 arcs got magnified into a problem of 28 nodes and 34 arcs (the extended network). Two factors determine the size of the problem: (i) number of nodes in the basic network and (ii) number of time periods in the planning horizon. It is learnt that the PDS has about two thousand nodes in the basic network and the planning horizon has about 150 days. Therefore, if we take days as the basic units of time, then we will have about 3,00,000 nodes and about 6,00,000 arcs (decision variables) in the extended network. These problems can be solved quite efficiently with the computing power available today. However, if there is need to reduce the size of the problem, we may

think of time units as weeks, and this will drastically reduce the size of the problem. Creating the input data for the extended network based on the basic network, the requirements and the water availabilities is very cumbersome and is very difficult to construct manually.

## 5. Software Development

Network models are one of the most widely used optimization tools. These have a wide range of applications ([1], [2], [10], [11]). Traffic systems, rail, road, shipping and airline systems are analyzed and optimized using network models. Transportation, transshipment and assignment problems are special cases of network models. Many software and computer systems are analyzed and optimized using network systems. Communication networks is another important application area of network models. Oil and natural gas production and distribution systems use network models. The elegance and efficiency with which network flow problems are solved have attracted many scientists to formulate a variety of problems as network flow problems. A number of commercial and public domain software packages are available to solve network flow problems. Since commercial software packages are expensive and the public domain software packages are not easily adoptable to our requirements, we thought it would be worthwhile developing our own package for solving network flow problems. As of now our software has two main components: (i) a code for solving a general minimum cost network flow problems and (ii) a code that will act as interface between our network solver and the users in the irrigation department. The latter takes the minimal inputs from the users, formulate the problem, prepare the input file for the solver, read the solution of the solver and present the results in customer specified formats. We wish to add a number of new modules to our software meant for other application areas.

We have adopted one of the most popular and efficient algorithms, namely the primal network simplex algorithm, for solving the minimum cost network flow problems. See [11] for a detailed description of the algorithm. We shall briefly outline some of the important aspects that we have considered in the development of our software. But before we do this let us take a quick look at the minimum cost network flow problem.

#### 5.1. Minimum Cost Network Flow Problem

A directed network consists of a set of nodes N = $\{1, 2, \ldots, n\}$  and a set  $A = \{(i_1, j_1), \ldots, (i_m, j_m)\}$  of m directed arcs. Each arc (i, j) is obtained by joining two nodes i and j from N. Here, i and j are called the tail and head nodes of the arc (i, j). Two subsets S and T of N are specified, S is called the set of source nodes and T is called the set of sink nodes. A certain commodity is to be supplied from nodes of S to nodes of T through the arcs of the network. Each node i in Sis assigned an integer  $a_i$  which means that i can supply at most  $a_i$  units of the commodity. Similarly, each node j in T is assigned an integer  $d_i$  which means that j requires  $d_i$  units of the commodity. Since the network is a directed network, arc (j, i), if exists in A, is treated as different from (i, j). Three numbers,  $l_{ij}$ ,  $k_{ij}$  and  $c_{ij}$ , are associated with each arc (i, j) in A. Here  $l_{ij}$ and  $k_{ij}$  are called the lower and upper bounds of the arc (i, j) and the quantity of the commodity transferred through the arc, called the flow in the arc, should be within the limits  $l_{ij}$  and  $k_{ij}$ . Next,  $c_{ij}$  is the cost of flow per one unit of commodity. Given these inputs, the minimum cost network flow problem is to determine the flows in the network arcs so that the total cost of transportation is a minimum and that the requirements of Tare met, availability constraints of S are not violated, and the flow restriction on the arcs are not violated. A mathematical statement of this problem can be stated as follows: Find the flows  $f_{ij}$ ,  $(i, j) \in A$  so as to

$$\label{eq:minimize} \begin{split} & \sum_{(i,\ j)\in A} c_{ij}.f_{ij} \\ & \text{subject to} \quad f(N,\ j) - f(j,\ N) = d_j \text{ for each } j\in T, \\ & \quad f(N,j) - f(j,N) \leq a_j \text{ for each } j\in S, \\ & \quad f(N,j) = f(j,N) \text{ for each } j\in N\setminus (S\cup T), \\ & \quad l_{ij} \leq f_{ij} \leq k_{ij} \text{ for each } (i,\ j) \in A, \\ & \text{and} \quad f_{ij} \geq 0 \text{ for all } (i,\ j) \in A. \end{split}$$

where  $f(N, j) = \sum_{i \in N: \ (i, \ j) \in A} f_{ij}$  and  $f(i, N) = \sum_{j \in N: (i, \ j) \in A} f_{ij}$ . The total availability of the material is equal to  $\sum a_i$  and the total demand is equal to  $\sum d_i$ . If  $\sum a_i = \sum d_i$ , then we call the network problem as a balanced network problem. We shall now illustrate the network model with an example.

**Example 2.** Consider the network with five nodes and seven arcs shown in Figure 6. The arc lower bound, upper bound and the unit costs are shown as triplets  $(l_{ij}, k_{ij}, c_{ij})$ .

The first step in solving this problem is to augment

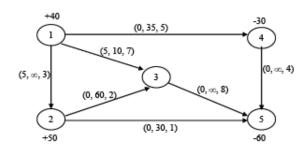


Fig. 6. A Network Problem with 5 Nodes and 7 Arcs

the network with two additional nodes 0 and (s+t+1)arcs, where s and t are the number of nodes in S and Trespectively, so as to make it a single source and single sink network problem. The arc between the two newly added nodes will be referred to as the main arc. We then convert the problem into a balanced network problem by defining suitable availability/requirement for the two new nodes if necessary. The augmented problem for the network problem in Figure 6 is shown in Figure 7. Like in the two-phase simplex method, a feasible solution to the network flow problem may be obtained by first considering the Phase-I objective function. We can obtain a feasible solution to a network flow problem, if one exists, using the augmented network in which all the unit costs (including the newly added arcs) are zero except for the main arc. The unit cost of the main arc is taken as 1. Since the augmented problem is feasible, we stop as soon as we get an optimal solution. If in the optimal solution, the flow in the main arc is zero, then this solution is a feasible solution for the original problem. On the other hand, if the flow in the main arc is positive, then it means that the original problem has no feasible solution. When the original problem has no feasible solution, the requirements of the nodes of T are partially fulfilled by the optimal solution of the Phase-I. In such situations, we switch over to the original costs, take zero as the cost for all the augmented arcs except the main arc. The cost of the main arc is treated as infinity. In our software, it is ensured that once the main arc becomes a nonbasic arc (that is, the flow in it becomes zero), it never enters the basis again.

## 5.2. Resolution of Cycling and Stalling

We use efficient data structures in our program to store and update the data in each iteration of the algorithm. Though degeneracy and stalling problems occur

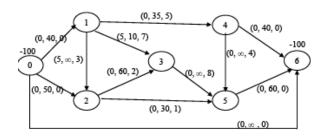


Fig. 7. The Augmented Network Problem

very rarely in real world applications (see [11] for details), a good software must take care of these issues. To avoid cycling due to degeneracy, we have implemented dropping arc choice rule developed by Cunningham ([4], [5]). If the starting feasible solution corresponds to a strongly feasible partition (see pp.327 [11]), then Cunningham's method does not allow cycling. Cunningham ([4]) has also suggested an efficient method of obtaining a solution corresponding to a strongly feasible partition from an arbitrary feasible solution. However, this part is yet to be implemented in our program. This is because we can easily obtain a solution corresponding to strongly feasible partition for the network problem arising out of water scheduling problem discussed in this report. Another rule suggested by Cunningham ([4]) to avoid stalling suggests that every arc be examined periodically and select it as entering arc if it is eligible at that time. This has been implemented in our program.

### 6. Concluding Remarks

Motivated by the acute water shortage problem faced by the farmers in the recent times, we have initiated this work on water distribution planning and scheduling in irrigation projects. Agricultural irrigation uses considerable volumes of water and efficient management of water resources is utmost important. We have used the Pennar Delta System of Nellore District in Andhra Pradesh, India, to build our model. The project concerns providing water distribution and scheduling for given requirements of the ayacut and given availabilities of water at various nodes of the irrigation network. The problem is formulated as a dynamic minimum cost network flow problem. We have developed a computer software to aid the users in irrigation department as a decision support system. This software has two main components the first of which is a network solver meant for solving general minimum cost network flow problems. The second component is an interface software between the network solver and the users in the irrigation department. We wish to broaden the scope of application of our software to many other areas of optimization.

Some parts of the western countries have been using the World Wide Web for efficient irrigation management practices. For a number of years, the North Dakota State University Extension Service has been using the Internet to deliver information, beginning with a Gopher server in the early 1990's and more recently with a World Wide Web server. To take full advantage of todays information technology and the computing power, we must quickly establish good database systems for effective planning and implementation.

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