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Article abstract

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Book Reviews

Uncertainty and Information: Foundations of Generalized Information Theory (a book review)

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Abstract

We present a review of the book by George J. Klir, Uncertainty and Information: Foundations of Generalized Information Theory, Wiley, Hoboken, New Jersey 2006, ISBN: 0-471-74867-6, 520 pp.

Key words: Key words: Uncertainty, information theory, imprecise probability, fuzzy sets.

1. Introduction

The foundation of information theory is usually connected with the name of Claude Shannon and a series of his papers from 1948. These papers started the development of the classical probability-based information theory. This theory approved its applicability to a wide diversity of theoretical problems as well to many specific areas. The "older brother" of Shannon entropy, Hartley measure of uncertainty (1928) is much less popular, sometimes even considered to be its special case. Nevertheless, both these measures of uncertainty (and therefore also the information measures based on them) are based on the "solid grounds" of classical (crisp) set theory and classical (additive) probability theory.

The emergence of new theories alternative to both classical set theory and probability theory gave rise to the necessity to build up a new, more general information theory. In 1980's and 1990's a lot of effort was made to do it, many researchers took part in this aspiration, nevertheless, the author of the book was one of the most active scholars. The reason of this effort was based on the idea that a generalized measure of uncertainty in a more general framework can play the role of Shannon entropy in probabilistic setting.

The result of this effort is this book, which, starting from the classical Hartley and Shannon measures of uncertainty, brings the current state of the art in generalized information theory. The generalization is twofold: the classical (crisp) sets are substituted by fuzzy sets and probability measures are replaced by monotone measures and imprecise probabilities of various kinds.

2. Contents

Chapter 1 is the introduction into the information theory in the general sense. The author stresses the significance of uncertainty, introduces uncertainty-based information and explains what can be understood under the notion generalized information theory. It also contains relevant terminology and notation and, as one can expect, an outline of the book.

Chapter 2 is devoted to classical possibility-based uncertainty theory. After the introduction of crisp (or two-valued) possibility functions, Hartley measure, its derivation, proof of its uniqueness and its basic properties are presented. Hartley-like measure of uncertainty for infinite sets is introduced and its properties are proven.

Chapter 3 is the probabilistic counterpart of Chapter 2. After a brief reminder of basic notions of classical probability theory it focuses on Shannon entropy, its derivation, axioms, proof of its uniqueness. As the literature on information theory based on Shannon entropy is extensive, only the most fundamental properties of Shannon entropy are presented here. The problems with the introduction of its counterpart for infinite sets

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are discussed in the end of this chapter.

Chapter 4 presents a general approach to monotone (non-additive) measures and imprecise probabilities. Different types of representation (Möbious representation, lower an upper probabilities, convex sets of probability distributions) and their mutual correspondence are described. It contains also sections on Choquet capacities and Choquet integral. The importance of imprecise probabilities is supported by numerous arguments.

Chapter 5, on the other hand, is devoted to special theories of imprecise probability. The main attention is devoted to possibility measures, Sugeno λ-measures, belief and plausibility measures and to so-called reachable interval-valued probability distributions. Other types of monotone measures are only briefly mentioned.

Chapter 6, the largest chapter of the whole book, presents measures of uncertainty and information in the frameworks listed in Chapter 5. It starts with the general discussion of this topic. Generalizations of Hartley measure in possibility theory, Dempster-Shafer theory and for convex sets of probability distributions are followed by the list of numerous attempts to generalize Shannon entropy in the framework of Dempster-Shafer theory, resulting in the definition of the so-called aggregate uncertainty. Two views to disaggregation of total uncertainty into two parts (non-specificity and conflict) are discussed.

In Chapter 7 one can find basic notions from standard fuzzy set theory, operations on standard fuzzy sets, fuzzy numbers and intervals, standard and constrained fuzzy arithmetic and fuzzy relations. Fuzzy logic is presented in its broad sense as a tool for modeling vague linguistic terms; truth-qualified and probability qualified fuzzy proposition are presented. The relation to approximate reasoning is also mentioned as well as nonstandard fuzzy sets. A section on fuzzy systems deals mainly with granulation and defuzzification.

Chapter 8 is devoted to fuzzification of uncertainty theories. The general aspects of fuzzification are followed by a section on measures of fuzziness and fuzzy-set interpretation of possibility theory. A simple fuzzification of classical probability theory based on substituting a classical event by a fuzzy event. and fuzzification of reachable interval-valued probability distributions are presented. In the end of the chapter an illustration of the fact that not every theory of uncertainty can be easily fuzzified is added.

Chapter 9 discusses four methodological principles of uncertainty (principle of minimum uncertainty, principle of maximum uncertainty, principle of requisite generalization, principle of uncertainty invariance) and their applicability. While the principles of minimum and maximum uncertainty can be applied within any each particular uncertainty theory, the remaining two facilitate transitions from one theory to another. The applicability of the principle of minimum uncertainty to simplification problems and conflict-resolution problems is presented. Principle of maximum uncertainty which is essential for any problems involving ampliative reasoning offers in generalized information theory five different functionals that can be maximized. Their choice depends mainly on the context of each application. Most attention is paid to the principle of uncertainty invariance in probability-possibility transformations, transformations between λ-measures and possibility measures, approximation of belief functions by necessity functions and in approximation of graded possibilities by crisp possibilities.

Chapter 10 consists of four sections: Summary and assessment of results in generalized information theory, Main issues of current research, Long-term research areas and Significance of generalized information theory. I think that their titles are self-explanatory.

3. Conclusion

As already mentioned above, this book contains the current state of the art in generalized information theory. Its significant advantage is that it is self-contained, it does not require any deeper mathematical background. It is clearly written (but for some typos) and a great number of illustrative examples help the reader to penetrate into the theory (and the teacher to test the knowledge of his/her students). It can be used not only as a reference book (and a textbook) in generalized information theory, but also as an introduction into the theory of imprecise probabilities. And I know, that I am not the only "imprecisionist" influenced at the very beginning by the author...

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