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See table of contents

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Article abstract

We present in this paper a theoretical approach that argues when one should opt for claims-made or for occurrence-based policies in liability insurance. Occurrence-based contracts cover the policyholder for losses incurred in a given year, no matter when the claims is actually reported in the future. In claims-made contracts, losses are covered in the year in which they are reported no matter when they occured in the past provided a claims-made insurance policy was valid then. The major difference between the two types of contract is thus that occurrence contracts are forward looking whereas claims-made contracts are retrospective. The goal of this paper is to analyze in what circumstances policyholders would prefer one contract of the other.

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Professional Liability Insurance Contracts: Claims Made Versus Occurrence Policies

by Martin Boyer and Karine Gobert

RÉSUMÉ

Nous présentons une approche théorique qui s'intéresse aux déterminants du choix de type de police d'assurance entre les contrats *Claims-made* et les contrats *Occurrence*. La grande différence entre les deux types de contrats est que le premier rend responsable l'assureur actuel pour des incidents passés qui sont rapportés aujourd'hui, alors que le second rend responsable l'assureur actuel pour les incidents aujourd'hui qui ne seront rapportés que dans le futur. Le but de l'article est justement de regarder dans quelles circonstances un assuré devrait préférer un contrat plutôt que l'autre.

Mots clés: Assurance responsabilité, assurance basée sur la date des réclamations, antisélection.

Classification JEL : G

ABSTRACT

We present in this paper a theoretical approach that argues when one should opt for claims-made or for occurrence-based policies in liability insurance. Occurrence-based contracts cover the policyholder for losses incurred in a given

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year, no matter when the claims is actually reported in the future. In claims-made contracts, losses are covered in the year in which they are reported no matter when they occured in the past provided a claims-made insurance policy was valid then. The major difference between the two types of contract is thus that occurrence contracts are forward looking whereas claims-made contracts are retrospective. The goal of this paper is to analyze in what circumstances policyholders would prefer one contract of the other.

Keywords: Liability insurance, claims made and reported, adverse selection.

JEL classification: G

1. INTRODUCTION

The introduction of *claims made and reported* (CMR hereafter) insurance policies has been one of the main innovations that resulted from the uncertainty surrounding the liability crisis of the late 1970s. As changes in the legal environment was an undiversifiable risk for insurers, the law of large numbers no longer applied completely to these insurance products, so that premiums needed to be increased to cover legal environment risk. Doherty (1991) concludes that the increased economic importance of mutual insurance companies resulted directly from this liability crisis. These contracts complemented traditional *occurrence-based* (OB hereafter) contracts whereby a policyholder is insured for losses that are incurred during the insurance policy year even if the loss is not reported for many more years. In contrast, CMR contracts insure policyholders for losses that are reported during the policy year even if the loss was incurred many years before (subject to a retrospective date or time limit). The apparent dominance of claims-made policy was such that even St. Paul Fire and Marine, a major medical malpractice insurer in the 1980s, switched its entire medical malpractice book of business to claims-made around that time. Posner (1986) anticipated that CMR policies would account for seventy to eighty percent of the medical malpractice insurance premium earned during 1985. Today, approximately 75% of medical malpractice insurance is being sold under a claims-made approcah.

The paper focuses on the difference in the policyholder's expected utility from each contract type. An underlying assumption we shall use is that the insurance market is competitive so that the premium paid is equal to each policyholder's expected loss. As a result, we are able to concentrate on the impact of the contract structure's differences rather than insurer profitability.

To better understand how the two types of contracts work, we concentrate on three features of the contracts: The way losses develop over the years, how insurance premiums are calculated, and the impact of risk aversion on the decision to purchase one contract or the other. Our paper presents two main results. First, we show that when everyone in the economy is risk neutral, then the only reason why insured agents would prefer the CMR contract is that the discount factor they apply to future cash flows is lower than the insurer's, which means that the present value of future cash flows is worth less to the agent than to the insurer. This result holds true whether we allow undiversifiable shocks to impact losses or the claim over time. Our second main result is that when insured agents are risk averse, then they are more likely to prefer the CMR contract over the OB contract when the tail of the loss is longer. This means that for short-tail lines, such as automobile and homeowner insurance, claims-made and reported insurance contracts do not dominate as much traditional occurrence based contracts than in long-tail lines such as medical and professional malpractice liability insurance.

Our theoretical approach offers predictions that one could bring to the data, such as the fact that for risk neutral agents, CMR contracts should be preferred to OB contracts when the insured agent has important liquidity constraints so that he discounts future cash flows much more than if he were not liquidity constrained.

The paper is organized as follows. We first present the economic importance of each type of insurance contract, how insured losses develop and how pure premiums should be calculated given these losses. We then present the problem that a risk neutral agent faces in an economy where there are no systematic shocks to the distribution of losses. We introduce shocks to the loss distribution in Section 4 and move to a risk averse agent in Section 5. Finally, we discuss the empirical predictions of the model that one could bring to the data and conclude in the last section of the paper.

2. ECONOMIC IMPORTANCE

Claims-made contracts are mostly popular in liability lines where the damage has been caused by an individual who exercises his profession. In particular CMR contracts are highly popular, or even the norm, is the case of medical malpractice liability insurance contracts, directors' and officers' liability insurance contracts and other situation of professional liability insurance contracts designed

to protect lawyers and architects in case of a professional error.[1] The rising importance of CMR policies has led the NAIC to include a separate aggregate report for lines of business where CMR policies are important starting in 1995. These lines of business are medical malpractice liability insurance, product liability insurance and other liability insurance, which include directors' and officers' insurance and other types of professional liability insurance. Table 1 presents the total of premiums earned in the United States for the case of the medical malpractice insurance line as a function of the type of contract from 1997 through 2006.

TABLE 1
PREMIUM EARNED (IN THOUSANDS OF CURRENT DOLLARS) IN MEDICAL MALPRACTICE INSURANCE BY TYPE OF CONTRACT, 1997-2006

year	Total premiums earned	OB contract premiums earned	CMR contract premiums earned	Percentage of total in CMR contracts
1997	5,032,842	1,441,057	3,591,785	71.4%
1998	5,128,893	1,465,654	3,663,239	71.4%
1999	5,267,617	1,591,712	3,675,905	69.8%
2000	5,351,526	1,861,044	3,490,482	65.2%
2001	5,780,544	1,718,908	4,061,636	70.3%
2002	9,157,351	2,430,379	6,726,972	73.5%
2003	8,302,736	2,496,678	5,806,058	69.9%
2004	8,784,556	2,145,038	6,639,518	75.6%
2005	8,629,529	2,065,908	6,563,621	76.1%
2006	10,140,990	2,355,646	7,785,343	76.8%

Source: Born and Boyer (2011) using the National Association of Insurance Commissioners annual data – Property and Casualty Insurers, Underwriting and Investment Exhibit. The table includes premium from all insurers reporting nonzero premiums in either type of medical malpractice insurance policies.

As we see, the market importance of claims-made contracts has risen over the years and accounts for close to 77% of the total premiums earned in the medical malpractice insurance line of business in 2006. This compares to 71% in 1997 and 70% in the early nineties according to Born and Boyer (2011). It is clear that CMR policies have increased in popularity and that they are gaining ground amongst policyholders in the medical malpractice insurance industry (see

Harrington, Danzon and Epstein, 2008, for a discussion of the market for CMR and OB policies).

It is not obvious that CMR contracts have increased in popularity in other lines of business where CMR contracts are available. In Panel A of Table 2 we present the total premiums earned by contract type for the medical malpractice, product liability and other liability lines of business for the years 2003 through 2006. We see that the popularity of claims-made contracts has increased relative to occurrence-based contracts only in the medical malpractice area; in the

TABLE 2
PREMIUM EARNED (IN MILLIONS OF CURRENT DOLLARS) AND NUMBER OF INSURERS BY TYPE OF CONTRACT IN PRODUCT LIABILITY AND OTHER LIABILITY LINES OF BUSINESS, 2003-2006

Panel A: Earned premium

Line of business	Contract	2003	2004	2005	2006
Medical malpractice	OB	$ 2,497	$ 2,145	$ 2,065	$ 2,356
	CMR	$ 5,806	$ 6,639	$ 6,564	$ 7,785
	%CMR/total	69.9%	75.6%	76.1%	76.8%
Product liability	OB	$ 2,165	$ 2,754	$ 2,936	$ 3,042
	CMR	$ 323	$ 391	$ 512	$ 527
	%CMR/total	13.0%	12.4%	14.8%	14.8%
Other liability	OB	$ 20,121	$ 23,698	$ 23,739	$ 26,407
	CMR	$ 12,323	$ 14,781	$ 15,402	$ 15,237
	%CMR/total	38.0%	38.4%	39.3%	36.6%

Panel B : Number of insurers

Line of business	Contract	2003	2004	2005	2006
Medical malpractice	OB	269	222	223	263
	CMR	332	365	354	394
Product liability	OB	560	551	534	543
	CMR	196	181	169	175
Other liability	OB	1222	1215	1226	1252
	CMR	575	584	571	577

Source: Born and Boyer (2011)

two other lines where claims-made contracts are available, there does not seem to be a large movement in the contract preferences of the policyholders. Even when we look at the number of insurance companies that provide each type of contract in these three lines as in Panel B of Table 2, we do not see insurers offering CMR contract being much more numerous relative to the number of insurers offering OB contracts. Another interesting comparison we can make between OB and CMR contracts is with respect to the loss ratio (losses incurred divided by premiums earned) for each type of contract.

Table 3 presents those loss ratios by contract type for the medical malpractice insurance industry.

TABLE 3
MEDIAN LOSS RATIO (LOSSES INCURRED DIVIDED BY PREMIUMS EARNED) IN THE MEDICAL MALPRACTICE INSURANCE INDUSTRY BY TYPE OF CONTRACT, 1997-2006

year	OB loss ratio	CMR loss ratio	CMR loss ratio vs. OB loss ratio
1997	0.732	0.770	Insignificant
1998	0.700	0.775	Significant*
1999	0.795	0.890	Significant*
2000	0.804	0.882	Significant*
2001	0.808	0.874	Significant*
2002	0.678	0.723	Significant*
2003	0.632	0.754	Significant*
2004	0.671	0.695	Significant*
2005	0.553	0.692	Significant*
2006	0.591	0.666	Significant*

Source: Born and Boyer (2011). The table includes only insurers with positive premiums. Losses incurred include defense and cost containment expenses.

It is interesting to note that CMR policies have a higher loss ratio than OB policies over the past ten years. And except for the year 1997, loss ratios for the two types of contract are significantly different from each other, with the loss ratio in the CMR line being higher than in the OB line. This suggests that underwriting profitability is lower in the CMR line than in the OB line. A natural question we can ask after looking at the economic importance of OB and CMR contracts and the type of losses they are designed to cover is why there

are two types of contracts that coexist in these insurance lines of business. What makes these markets attractive for both CMR and OB contracts to coexist whereas the traditional OB contract is the norm in other personal lines?

3. RISK NEUTRAL AGENTS AND SHOCKLESS ECONOMY

We now suppose that a risk neutral individual lives for K periods and is exposed to some risk during the first $T + 1$ periods, that is, the active periods of his life. In period $t \in [0,T]$ the agent faces a potential loss $\mathcal{L}_t$ with an invariant distribution $f(\cdot)$ and mean $E(\mathcal{L}_t) = L$. We suppose that there is no change in loss development patterns $(\{\alpha_t\}_0^n)$ over time and that any loss is fully developed before the individual dies, $T + 1 \leq K - n$.

Because the distribution and development of the loss is the same in every period, the OB premium will be the same in every period. The agent can, therefore, be insured with a series of OB contracts such that he pays a premium $\overline{P}_t$ in every period t from 0 to T and nothing after that, with

$$\overline{P}_t = \overline{P} = \sum_{j=0}^{n} \delta_I^j \alpha_j L.$$

In the case of CMR policies, the premium that is paid today must cover losses that are paid today, no matter when such a loss was incurred. In the first year of the CMR policy (at $t = 0$), only first year's reported losses need to be insured. This means that the first year's premium is given by $\hat{P}_0 = \alpha_0 L$. In the second year, at time $t = 1$, the premium is given by $\hat{P}_1 = \alpha_0 L + \alpha_1 L$; the first term is the amount that will be paid to cover losses that are incurred in period $t = 1$, whereas the second term is the amount that will be paid to cover losses that were incurred in period $t = 0$, but are not reported until period $t = 1$. At time $t = 2$, the premium is $\hat{P}_2 = (\alpha_0 + \alpha_1 + \alpha_2)L$. Before period n, premium is growing every year to account for the growing number of potential past losses. In period $t = n$, the period 0 loss has fully developed and the premium is

$$\overline{P}_t = \overline{P} = \sum_{j=0}^{n} \delta_I^j \alpha_j L.$$

After the first n years of the agent's professional life and until the end of his career, he can buy each year a CMR contract insuring

for loss reports from the n preceding years. Premium $\hat{P}_t = \hat{P}_n$ is then paid so long as the individual is working, for periods $t \in \{n,T\}$.

After retirement the individual still needs to purchase insurance to cover losses that were incurred in the past, but have not been reported yet. Such an individual does not need to purchase insurance for losses that are to be incurred this year since he is no longer exposed to such losses. In period $T + 1$, his premium then starts to decrease and becomes

$$\widehat{P}_t = \sum_{j=t-T}^{n} \alpha_j L, \qquad t = T+1,\cdots,T+n$$

until $t = T + n$. After that date, he no longer pays any premium ($\hat{P}_t = 0$ for all $t \geq T + n + 1$).

If individuals actualize future premiums at rate r_A invariant over time so that their discount factor is given by $\delta_A = \dfrac{1}{1+r_A}$, then the present value of all premiums paid over the individual's lifetime with an OB insurance policy is given by

$$\overline{\Pi} = \sum_{t=0}^{T} \delta_A^t \overline{P} = \sum_{t=0}^{T} \delta_A^t \sum_{i=0}^{n} \delta_I^i \alpha_i L$$

In the case of a CMR insurance policy, the present value of the premiums paid is

$$\widehat{\Pi} = \sum_{t=0}^{T+n} \delta_A^t \widehat{P}_t = \sum_{t=0}^{n} \delta_A^t \sum_{i=0}^{t} \alpha_i L + \sum_{t=n+1}^{T} \delta_A^t \sum_{i=0}^{n} \alpha_i L + \sum_{t=T+1}^{T+n} \delta_A^t \sum_{i=t-T}^{n} \alpha_i L.$$

Comparing $\overline{\Pi}$ with $\hat{\Pi}$, we can state the following proposition.

Proposition 1 : *If the loss development pattern* $(\alpha_0,\ldots,\alpha_n)$ *and the distribution of the loss do not change over time, then risk neutral policyholders will prefer a CMR contract to an OB contract if and only if* $\delta_I > \delta_A$ *(or equivalently,* $r_I < r_A$*).*

Proof. All proofs are relegated to the Appendix.

The difference between the two premium profiles stands in their timing and this matters for a risk neutral agent only if different discount factors can be affected to the payments. Suppose we look at the loss that is incurred in period 0. The OB premium for this loss discounts future potential losses at the insurer's rate: $\overline{P}_0 = \sum_{t=0}^{n} \delta_I^t \alpha_t L$. In the case of the CMR contract, the insurer receives a premium in each period for the payment he is likely to make in the same period. The insurer's discount rate does not influence the CMR premium whereas

the agent actualizes future premiums using discount factor δ_A. This means that the CMR contract cost perceived by the agent is $\hat{\Pi}_0 = \sum_{t=0}^{n} \delta_A^t \alpha_t L$. Therefore, an agent will prefer the CMR contract to the OB contract if and only if $\delta_I > \delta_A$ (i.e., if and only if $r_I < r_A$).

We can infer from this proposition that if the insurer and the policyholder use the same discount rate, then the risk neutral policyholder will be indifferent between the two policies. If the interest rate used by the agents (r_A) to discount future cash flows is greater then the insurer's interest rate (r_I), then the agent will surely prefer a CMR policy to an OB policy. As a result, myopic or impatient agents prefer to purchase CMR policies; because they value the present proportionally more than the future, these policyholders prefer to pay lower premiums in the short run even if it means paying more in the long run. This allows us to make our first prediction.

Prediction 1. For losses that are not influenced by external shocks and for risk neutral policyholders, a CMR policy will be preferred to an OB policy if and only if the insured's cost of capital is greater than the insurer's cost of capital.

4. SHOCKS TO THE LOSS DISTRIBUTION

Proposition 1 assumed that the distribution of losses was invariant in time. Doherty (1991) claims, however, that it is because of the uncertainty in future losses that insurers started offering CMR policies. We explore this possibility in this section. We introduce periodic shocks affecting the loss distribution. We must then distinguish between two cases. If the distribution of the shock is known to all so that the expected value of the shock is the same in period 0 as in the period where the shock occurs, there is no uncertainty on the loss distribution, only an anticipated trend is the expected loss. On the other end, if the distribution of the shock is unknown, the expected value in period 0 of the shock in t, is not the same as the expected value in t of the shock in t. In this case, there is uncertainty on future loss distributions since shocks are unanticipated. We show that whether the shock is anticipated or not has no impact on the risk-neutral comparative evaluation of the contracts.

4.1 Anticipated shocks to the loss distribution

Let us assume that the claim paid by the insurer can evolve independently of the event because, for instance, the size of the claim

is related to the regulatory environment in place at the moment the claim is filed rather than the moment the loss is incurred. The loss distribution is the distribution of the claim regardless of the period in which the event took place. That information is known to the policyholder and the insurer. The shocks to the loss distribution are anticipated in the sense that $E(\mathcal{L}_{t+i})$ is invariant, that is, there can be shocks to the loss distribution but the expected value of these shocks are perfectly known from period 0 on.

An OB contract signed in period t takes account of future expected claims that may arise following an event in t. The OB premium, then, is forward looking:

$$\overline{P}_t = \sum_{i=0}^{n} \delta_I^i \alpha_t E(\mathcal{L}_{t+i}).$$

On the other hand, in a CMR contract, the premium in period t only depends on the loss claimed in period t regardless of when in the past the event occurred. The premium writes:

$$\widehat{P}_t = \sum_{i=k}^{l} \alpha_i E\left(\mathcal{L}_t\right),$$

where $k = 0, l < n$ if $t < n$, $k = 0, l = n$ if $n \leq t \leq T$ and $k = t - T, l = n$ if $t > T$.

Premiums are no longer constant in either form of contract. Making a choice between CMR or OB contract in period t, an agent considers the difference in premiums. The next Proposition establishes that from period t on, only discounting can make a difference between the two types of contracts.

Proposition 2 : *If the distribution of loss changes over time, risk neutral policyholders will prefer a CMR contract to a OB contract if and only if* $\delta_I > \delta_A$ *(or equivalently,* $r_I < r_A$*).*

Suppose a single event occurring in period t. The OB premium is paid only once and covers all future claims related to this loss:

$$\overline{P}_t = \sum_{i=0}^{n} \delta_I^i \alpha_t E(\mathcal{L}_{t+i}).$$

The sequence of premiums that will be paid using a CMR contract to cover for the period t loss is given by

$$\hat{P}_{t+i} = \alpha_t E(\mathcal{L}_{t+i}), \quad i = 0, \cdots, n.$$

In period t, the agent correctly anticipates that the CMR premium will be $\hat{P}_{t+i}$ in $t + i$. There is, then, no uncertainty on future premiums, neither in OB nor in CMR arrangements. The discounted sum in t of future CMR premiums related to an event in t is, then,

$$\sum_{i=0}^{n} \delta_A^i \hat{P}_{t+i} = \sum_{i=0}^{n} \delta_A^i \alpha_t E(\mathcal{L}_{t+i})$$

If $\delta_I = \delta_A$, this amounts to the same. It is clear that an agent prefers a CMR arrangement only if his discount factor is lower than the insurer's one. With equal discount factor, a risk neutral agent is indifferent between paying $\overline{P}_t$ now or anticipating to pay a sequence of premiums $\hat{P}_{t+i}$, $i = 0,\cdots,n$ that he discounts.

As long as the distribution of the shocks is known, there is no uncertainty and the expected value of a period t loss is correctly anticipated in period 0 by both the insurer and the insured. Proof of Proposition 4.1 makes it obvious, once the terms are rearranged to highlight the expected discounted sums of premiums, that only the discount factors matter in comparing the two premium profiles. Accounting for the associated future loss distribution of losses does not change a risk neutral agent's evaluation of the intertemporal value of each type of contract.

4.2 Unanticipated shocks to the loss distribution

In this subsection, we explore the case of an uncertain distribution of future losses to see if that introduces a difference between the two types of contracts. Uncertainty means that the expected value of a period t loss is not the same if it is evaluated in period t with the information available in period t as if it is evaluated in an earlier period. Put differently, we will assume that even if agents know that the expectation they have of future losses is wrong, they cannot improve it. In that case, OB premiums, that are forward looking, rely on imperfect information about future losses.

Let us denote $E_t(\mathcal{L}_{t+i})$, $i = 0,\cdots,n$, the expected value of a claim in $t+$ i conditional to information available in t. Unanticipated shocks to the loss distribution imply that the distribution $\mathcal{L}_t$ varies from period to period so that $E_t(\mathcal{L}_{t+i}) \neq E_\tau(\mathcal{L}_{t+i})$.

The OB premium for an event incurred in period t is paid in t and equal to

$$\overline{P}_t = \sum_{i=0}^{n} \delta_I^i \alpha_i E_t(\mathcal{L}_{t+i}).$$

If the agent decides to enter a sequence of CMR contracts, he should expect to pay a sequence of premiums $\hat{P}_{t+i}$, $i = 0,\cdots,n$ such that

$$\hat{P}_{t+i} = \alpha_i E_{t+i}(\mathcal{L}_{t+i}).$$

The best predictor an agent can use in period t to evaluate $E_{t+i}(\mathcal{L}_{t+i})$ is $E_t(E_{t+i}(\mathcal{L}_{t+i}) = E_t(\mathcal{L}_{t+i})$. Hence, in period t, the policyholder's expected discounted sum of future premiums paid in a CMR arrangement is

$$E_t\left(\sum_{i=0}^{n}\delta_A^i\,\widehat{P}_{t+i}\right)=E_t\left(\sum_{i=0}^{n}\delta_A^i\alpha_i E_{t+i}(\mathcal{L}_{t+i})\right)=\sum_{i=0}^{n}\delta_A^i\alpha_i E_t(\mathcal{L}_{t+i}).$$

Again, the discounted value of insurance evaluated in t is the same in both contracts if $\delta_A = \delta_I$. However, the insurer and the insured know that the OB premium is computed with imperfect information. The OB insurer is the bearer of this error's consequences since he is the one to pay for potential future losses the true expected value of which he does not know. A CMR contract shifts the burden of uncertainty to the insured who cannot compute expected future premiums he will have to pay in the future to insure a current event. However, a CMR type of contract is a better way of dealing with uncertainty in the sense that it allows to wait for accurate information before premiums are computed. In standard property/casualty insurance contracts, a premium is paid each period and incorporates all new information arrived in the period. It is, then, surprising that OB contracts still exist that lock the insurer into future obligations after premiums have been paid once and for all. This leads to our second prediction.

Prediction 2. For losses that are influenced by unanticipated shocks a CMR policy should be preferred to an OB policy because it allows to account for relevant information at the time it accrues.

5. RISK AVERSION

Given that OB and CMR contracts offer premiums that do not differ in present value, it follows that if insured agents discount the future at the same rate as the insurers, an important motive for choosing one of the contracts must be that they are somehow risk averse. A risk averse agent endowed with a concave utility function has a preference for the smoothing of his consumption. Since the difference between CMR and OB contracts is all about the timing of cash flows, a risk averse agent may prefer one contract over the other because of this smoothing effect.

Suppose an agent has a constant wealth Y per period. His consumption in period t is $Y - P_t$ where P_t is the premium paid in t to buy an insurance contract. The agent values consumption with a concave utility function $u(Y - P_t)$ with $u' > 0$ and $u'' < 0$. We assume that the distribution of losses is invariant such that $\mathrm{E}(\mathcal{L}) = L$. The agent now compares intertemporal utilities under each contract. With OB and CMR premiums respectively equal to $\overline{P}_t$ and $\hat{P}_t$ as defined in section 3, the agent's intertemporal utility over his entire life is

$$U_{OB} = \sum_{\tau=0}^{T} \delta_A^{\tau} u\left(Y - \sum_{i=0}^{n} \delta_I^i \alpha_i L \right) + \sum_{\tau=T+1}^{K} \delta_A^{\tau} u(Y)$$

with a sequence of OB contracts. With a sequence of CMR contracts, his intertemporal utility from period 0 on is:

$$U_{CMR} = \sum_{\tau=0}^{n} \delta_A^{\tau} u\left(Y - \sum_{i=0}^{\tau} \alpha_i L \right) + \sum_{\tau=n+1}^{T} \delta_A^{\tau} u(Y - L)$$

$$+ \sum_{\tau=T+1}^{T+n} \delta_A^{\tau} u\left(Y - \sum_{i=\tau-T}^{n} \alpha_i L \right) + \sum_{\tau=T+n}^{K} \delta_A^{\tau} u(Y).$$

The CMR sequence of premiums is characterized by payments made over a longer period compared to the OB sequence of contracts. In the first periods of one's professional life, the CMR contract charges lower premiums. The agent's consumption in an early period $\tau < n$ is higher under a CMR arrangement for all τ such that

$$\sum_{i=0}^{\tau} \alpha_i \leq \sum_{i=0}^{n} \delta_I^i \alpha_i.$$

After that date, and for the remainder of the agent's life, the CMR premium is never lower than the OB premium. Not only is the CMR premium equal to $\hat{P} = L > \sum_{i=0}^{n} \alpha_i \delta_I^i L = \overline{P}$ from the moment losses become entirely developed until the retirement date, but, more importantly, a CMR contract requires premiums paid after the agent has ceased any professional activity that could generate a loss. That is, for n periods after period T + 1, consumption continues to be lower under a CMR because the agent still pays a premium under a CMR contract whereas he no longer pays anything under an OB contract.

The agent's preference for one or the other of the two types of contracts shows in the difference between the intertemporal utilities. Let us denote $DU = U_{CMR} - U_{OB}$ the difference between the lifetime utility an agent gets from a CMR contract and the lifetime utility an agent gets from an OB contract. A risk averse agent will prefer a CMR contract if and only if DU is positive. We have:

$$DU = \sum_{\tau=0}^{n-1} \delta_A^{\tau} \left(u\left(Y - \sum_{i=0}^{\tau} \alpha_i L \right) - u\left(Y - \sum_{i=0}^{n} \alpha_i \delta_I^i L \right) \right)$$

$$+ \sum_{\tau=n}^{T} \delta_A^{\tau} \left(u(Y - L) - u\left(Y - \sum_{i=0}^{n} \alpha_i \delta_I^i L \right) \right)$$

$$+ \sum_{\tau=T+1}^{T+n} \delta_A^{\tau} \left(u\left(Y - \sum_{i=\tau-T}^{n} \alpha_i L \right) - u(Y) \right)$$

Since the evaluation of consumption with a concave utility function implies a preference for smoother consumption paths, and since the agent is partly myopic and discounts future utility with factor δ_A, the natural smoothing advantage of a CMR contract over an OB is increased if δ_A is relatively low. Moreover, this advantage is also greater, the longer the delay (τ) before $\sum_{i=0}^{\tau}\alpha_i$ becomes larger than $\sum_{i=0}^{n}\delta_I^i\alpha_i$, that is, the longer the delay before the CMR premium $\hat{P}_\tau = \sum_{i=0}^{\tau}\alpha_i L$ becomes larger than the OB premium $\overline{P}_\tau\sum_{i=0}^{n}\alpha_i\delta_I^i L$.

This delay has two major components: The development pattern $(\alpha_0,\cdots,\alpha_n)$ and the discount rates (δ_I, δ_A). These two component are key to comparing the CMR sequence of contracts to the OB sequence of contracts. The direct effect of the size of the tail (the value of n) on this delay is ambiguous, however. On one hand, the longer the tail, the longer the number of early periods over which the CMR premium is likely to be lower than the OB one. On the other hand, the thicker the tail (the higher the value of α_i for distant periods i), the stronger the discounting effect that reduces the OB premium.

Hence, we can summarize the CMR advantage as follows. The difference DU is more likely to be positive

1. When δ_A is low;
2. When δ_I is low and the loss has a short and thin tail;
3. And when δ_I is high and the loss has a long and thick tail.

Again, discounting is a core determinant of each contract evaluation. The differential in discount factors is, however, no longer sufficient to explain an agent's preference for one type of insurance contract or the other. Because of its impact on the timing of consumption, the shape of the development pattern cannot be ignored. As a result, we would like to isolate this timing effect. To do so, we shall assume that all players in the economy have the same discount rate so that $\delta_A = \delta_I = \delta$. This allows us to remove the discount factor differential as the source of the agent's preference for one contract rather than the other and to concentrate exclusively on the risk aversion effect. In a first approach, we simplify the problem in considering a single event ($T = 1$). Our second simplifying approach will be to reduce the framework to a short tail loss whereby the loss development pattern will be limited to only two periods ($n = 1$). In the last subsection, we illustrate our results with some numerical computations under these two combined approaches ($T = 1$ and $n = 1$).

5.1 A pure smoothing effect

With a single event framework ($T = 1$) and equal discount factors, the difference in utility of purchasing the CMR sequence of contracts over the OB sequence of contracts writes

$$DU = \underbrace{u(Y-\alpha_0 L)-u(Y-\sum_{i=0}^{n}\alpha_i\delta^i L)}_{DU^+}+\underbrace{\sum_{i=1}^{n}\delta^i\left(u(Y-\alpha_i L)-u(Y)\right)}_{DU^-}$$

As we stated earlier, the advantage of the CMR sequence of contracts is that it offers a better smoothing of the premiums whereas the advantage of the OB sequence of contract is that the insurer's discounting decreases the OB premium in the first periods. The observation of DU for any pattern $(\alpha_0,\cdots,\alpha_n)$ leads to the following proposition

Proposition 3 : *For a single event in period 0 and no event afterwards and a development pattern of any length, a risk averse agent with the same discount factor as the insurer prefers a CMR contract to a OB contract if there is no discounting* ($\delta=1$). *He is, however, indifferent between the two contracts if the future is totally discounted* ($\delta=0$).

Obviously, if neither the agent nor the insurer takes account of the future, the OB premium is reduced to $\overline{P} = \alpha_0 L$ and future payments are not considered in the intertemporal utility. In that case, the intrinsic difference between the two types of contracts is ignored together with the timing of payments. On the other hand, when the future is not discounted ($\delta=1$), higher future CMR premiums are completely taken into account. Proposition 5.1 establishes that preferences for a smooth consumption is sufficient to make a risk averse agent prefer a CMR contract over an OB contract. This argument is important and asserts that the key explanation for the choice of a CMR contract can be a strong preference for consumption smoothing over the agent's life.

When there is discounting, however, the first period OB premium is lowered. This lowering of the OB premium could be such that the benefit of a lower initial period premium can overcome the CMR's smoothing advantage so that a risk averse agent could prefer the OB contract. The CMR's pure advantage is a first period advantage: $DU^+ = u(Y-\alpha_0 L) - u(Y-)\sum_{i=0}^{n}\delta^i\alpha_i L > 0$. For a given pattern $(\alpha_0,\cdots,\alpha_n)$, DU^+ continuously increases with δ. However, DU^+ is greatly dependent on the size of α_0 and the dispatching of the $\alpha_i, i = 0,\cdots,n$ in the simplex $\sum_{i=0}^{n}\alpha_i = 1$.

Another advantage of the sequence of OB insurance contracts is an absence of premium in the last n periods of the agent's life so that $DU^- = \sum_{i=1}^{n}\delta^i(u(Y-\alpha_i L) - u(Y)) < 0$. Obviously, in absolute terms, this particular advantage of the OB contract increases with δ. This

means that the effect of δ on DU is ambiguous since DU^+ and DU^- become larger, in absolute terms, as δ becomes larger. This means that the impact of a change in the discount factor on an agent's preference for a CMR contract over an OB contract will depend on the loss development pattern $(\alpha_0,\alpha_1,\cdots,\alpha_n)$. Consequently, the structure of the tail is an important determinant of an individual's insurance purchasing decision between an OB and a CMR contract. The next proposition presents how an agent's decision is affected by the development pattern and the discount factor.

Proposition 4 : *There is a* $\delta^* \in [0,1]$ *such that, for a given development pattern* $(\alpha_0,\cdots,\alpha_n)$*, the CMR advantage DU is increasing in* δ *for* $\delta > \delta^*$.

Discounting remains an important factor for the choice of a type of contract when the insured is risk-averse, even when insurer and insured share the same discount factor. And even though a risk averse agent values the timing of the insurance payments he makes, the discount factor has an ambiguous impact on the advantages of purchasing one type of contract over the other. We know from Proposition 5.1 that a sequence of CMR contracts is preferred when $\delta = 1$, and we know from Proposition 5.1 that this advantage is increasing when measured close to $\delta = 1$. We can, then, conclude that there are values of δ in the neighborhood of 1 for which a CMR contract is preferred by a risk averse agent. No other general pattern can be extracted from our analysis except for the fact that the advantage of the CMR contract is increasing in the discount factor for large enough values of this discount factor. The impact for smaller values of δ is ambiguous, however, because it depends on the distribution of the α_i. The α_i determine the length of time during which CMR premiums must be paid after retirement. The longer n, the higher the CMR disadvantage due to the extended length of payments.

In the next subsection, we concentrate on the effect of the development pattern. We compare thick and thin tails in a case where the loss fully develops in two periods.

5.2 A two-period insurance line

We concentrate in this section on a simplified two-period development pattern to isolate results related to the thickness of the tail rather than it length. We will then have a situation in which $n = 1$ so that $\alpha_1 = 1 - \alpha_0$. For a potential event in each of the T-period agent's professional life, the CMR net advantage (i.e., the utility of having a CMR contract minus the utility of having an OB contract) writes:

$$DU = u(Y-\alpha_0 L) - u(Y-(\alpha_0+\delta(1-\alpha_0))L) + \sum_{t=1}^{T}\delta^t\left(u(Y-L)-u(Y-(\alpha_0+\delta(1-\alpha_0))L)\right) + \delta^{T+1}\left(u(Y-(1-\alpha_0)L)-u(Y)\right)$$

Only the first period utility is greater in a CMR contract with a two period line. After the initial period $t = 0$, the OB premium is consistently lower than the CMR premium. Hence, if *DU* were to be positive, it would mean that the difference $u(Y - \alpha_0 L) - u(Y - (\alpha_0 + \delta(1-\alpha_0)L)$ must be high enough to compensate for all subsequent discounted negative differences. A first prediction is that a low α_0 combined with a high δ would increase the OB premium and, then, the advantage associated with the CMR contract in the first period. The problem is that a low α_0 combined with a high δ also increases the last period OB advantage: $\delta^{T+1}(u(Y - (1 - \alpha_0)L) - u(Y)) < 0$. A low discount factor δ (particularly a low δ_A) on the other hand will decrease the weight of these negative future differences. The first period advantage of the CMR increases when α_0 decreases, this is all the more true if δ is high (particularly if δ_I is large, to increase the OB premium).

We can , then, expect that in the case of a short tail insurance line of business (as in a two-period line), a decrease of the tail, as measured by an increase in α_0, will increase the advantage of the OB contract over the CMR. This is stated in the next proposition.

Proposition 5 : *If the tail of the loss is short* (n = 1)*, then, at the margin,*

- *if the tail is thick* ($\alpha_0 \leq (1 - \delta)/(2 - \delta)$)*, the CMR sequence of contract becomes more advantageous for a risk averse agent when the tail becomes thinner* (α_0 *increases*)*;*
- *if the tail is thin* (α_0 *close to* 1)*, the OB sequence of contracts becomes more advantageous for a risk averse agent when the tail becomes even thinner* (α_0 *increases*).

This allows us to infer that the CMR advantage is not monotonic in the thickness of the tail, at least based on the differences between the insured agent's intertemporal utilities under each form of contract. There is in fact a value $\alpha_0^* \in (\frac{2-\delta}{1-\delta}),1]$ around which the CMR advantage *DU* is at its maximum.

5.3 Numerical computations

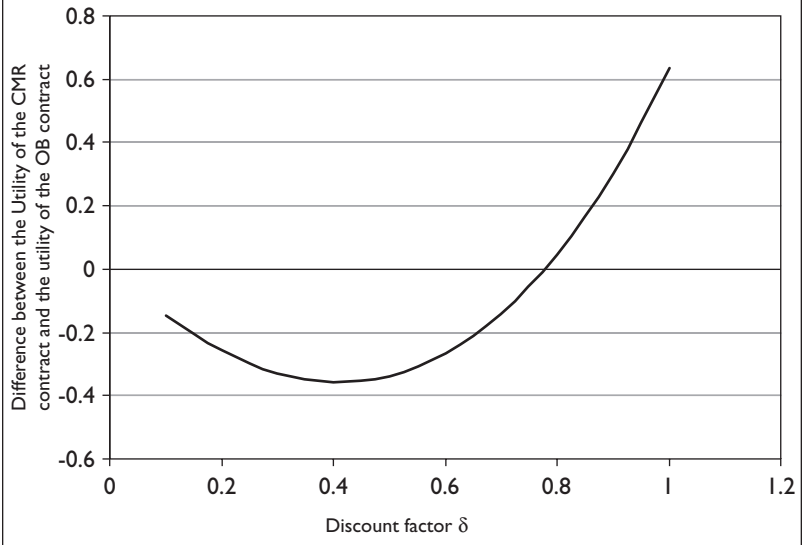
It has not been possible to prove that the CMR advantage *DU* is consistently positive except when $\delta = 1$. To examine the sensitivity of intertemporal utilities under each type of contract to the variations of δ and α_0, we ran some numerical computations to illustrate our results. For a standard CRRA utility function $u(Y) = \frac{Y^{1-\gamma}}{1-\gamma}$ with $\gamma = \frac{1}{2}$, and a single event ($T = 1$) developing over two periods ($n = 1$) so that $\alpha_1 = 1 - \alpha_0$, we compute

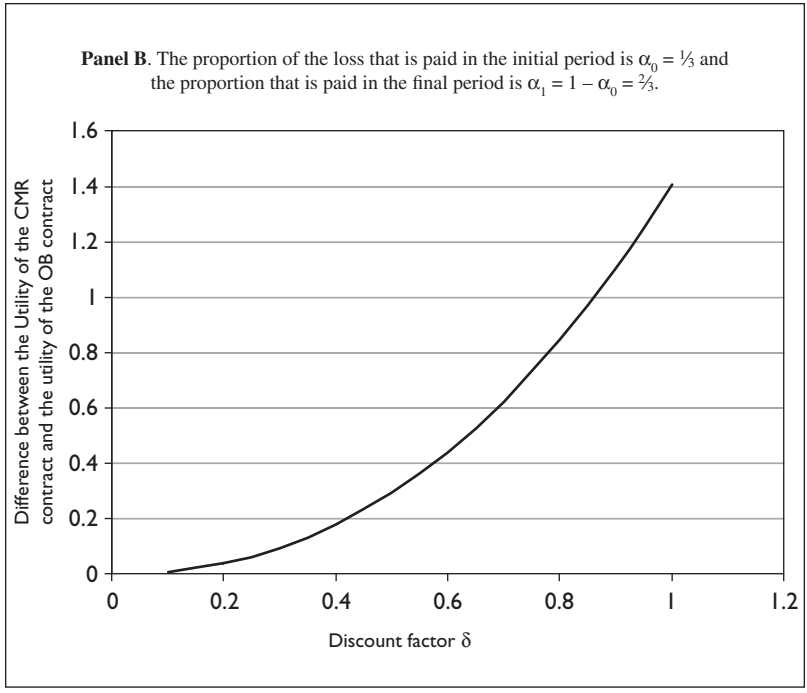
$$DU = (u(Y - \alpha_0 L) + \delta u(Y - \alpha_1 L)) - (u(Y - \alpha_0 L - \delta\alpha_1 L) + \delta u(Y))$$

DU depends on the values of δ and α_0. For a given α_0, Panel A

FIGURE 1
EXPECTED UTILITY OF HAVING A CLAIMS-MADE CONTRACT COMPARED TO AN OCCURRENCE CONTRACT AS THE DISCOUNT RATE (δ) VARIES, WITH AN AGENT WHO LIVES ONLY THROUGH ONE POSSIBLE EVENT ($T = 1$), WHEN LOSSES FULLY DEVELOP OVER TWO PERIODS ($n = 1$), DEPENDING ON THE PROPORTION OF THE LOSS PAID IN THE INITIAL PERIOD, α_0.

Panel A. The proportion of the loss that is paid in the initial period is $\alpha_0 = \frac{1}{9}$ and the proportion that is paid in the final period is $\alpha_1 = 1 - \alpha_0 = \frac{8}{9}$.





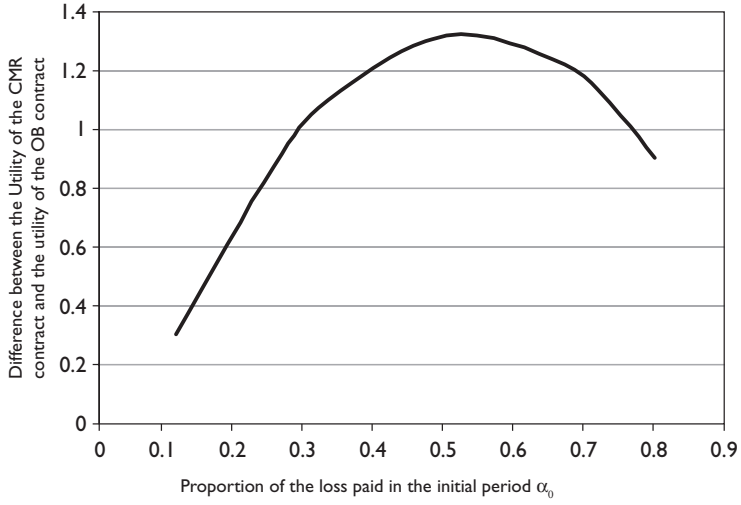
of Figure 1 shows that *DU* is U-shaped in δ if α_0 is small (strictly smaller than $1/3$ in our computations).

In our computations, *DU* can be negative and decreasing for low values of δ, increasing and then positive for higher values of δ. This illustrates Propositions 3 and 4. When α_0 increases, the value of δ for which *DU* is minimum decreases. For $\alpha_0 \geq 1/3$ as in Panel B of Figure 1, *DU* is positive for any value of δ, which means that a claims-made contract should be preferred by all agents who are exposed to a loss whereby more than one third of the losses are paid in the initial contract period.

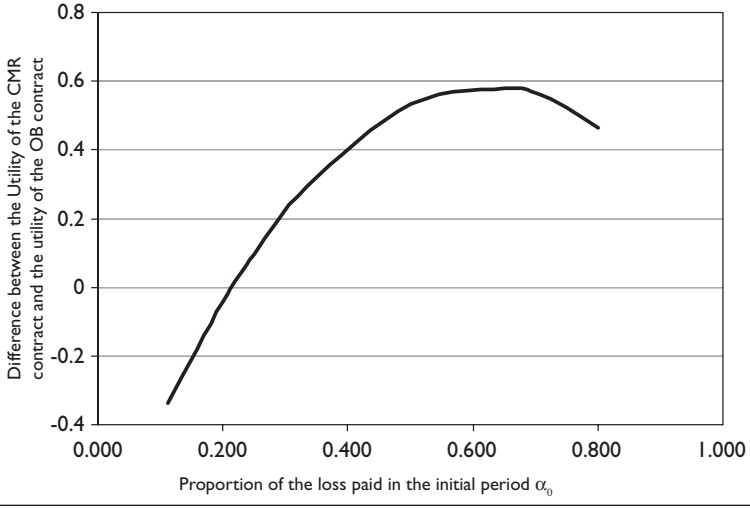
In Figure 2, we let the proportion of the loss paid in the initial period vary while keeping all other parameters constant. We see in both Panel A (where the discount factor is high) and Panel B (where the discount factor is low) that for a single event, the relationship between *DU* and α_0 is that of an inverted parabola. This illustrates Proposition 5. For high values of δ as in Panel A, we see that the agent is always better of with a CMR contract than an OB contract, no matter what the loss development pattern is. Proposition 5 says that *DU* is increasing in α_0 for $\alpha_0 < (1 - \delta)/(2 - \delta) = 0{,}09$, and decreasing for α_0 close to one. This holds true in this Panel A.

FIGURE 2
EXPECTED UTILITY OF HAVING A CLAIMS-MADE CONTRACT COMPARED TO AN OCCURRENCE CONTRACT AS THE PROPORTION OF THE LOSS THAT IS PAID IN THE INITIAL PERIOD (α_0) VARIES, WITH AN AGENT WHO LIVES FOR TWO PERIODS (T = 2), WHEN LOSSES FULLY DEVELOP OVER TWO PERIODS (n = 1) AND THE DISCOUNT RATE IS RELATIVELY HIGH (δ = 0.9).

Panel A. The discount factor is relatively high (δ = 0.9).



Panel B. The discount factor is relatively low (δ = 0.5).



For low values of δ as in Panel B, however, *DU* is negative for low values of α_0, but still increasing in α_0 for $\alpha_0 < (1 - \delta)/(2 - \delta) = 0{,}33$. Hence, agents are better off purchasing an OB contract when the proportion of the loss that is paid in the initial period is low and the agent discounts the future a lot. As α_0 increases, *DU* eventually becomes positive, which means that the CMR contract becomes preferred to the OB contract, and eventually reaches a maximum positive value. The advantage the CMR contract over the OB contract then decreases, but remains positive until $\alpha_0 = 1$. In our computations for this simplified framework where the development pattern is short, the value of *DU* is positive for most combinations of α_0 and δ.

Finally, we find that the CMR advantage decreases when the number of possible events, *T*, increases. With more than one potential event, that is when $T > 0$ and the agent insures for all his professional life, the increasing weight of the discounted OB premium plays for the OB contract and *DU* is more often negative.

6. CONCLUSION

The goal of this paper was to compare the relative efficiency of two types of insurance contracts offered to agents who seek financial protection against lawsuits brought upon them as a result of accidents while exercising their professional activities. These two types of contracts are known as the traditional and well-known *occurrence based* insurance contract, whereby insured agents are covered for losses that their incur in the policy year no matter when the claim is filed in the future, and the *claims-made and reported* insurance contract, whereby insured agents are covered for losses that are reported during the policy year no matter when the losses was incurred in the past. Claims-made contracts are popular in liability lines and account for 75% of the total earned premiums in the medical malpractice liability insurance line of business. At the same time, claims-made contracts are mostly inexistent in property insurance lines. Claims-made contract are also the default type of contract in the case of directors' and officers' liability insurance and many other professional liability insurance coverages. And although claims-made contracts also exist to cover product liability losses, they are much less popular than the traditional occurrence based contract since only 15% of product liability insurance contracts are claims-made. We were then faced with the following two empirical questions for which we wanted to build a model that explained the coexistence of the two types of contract:

- Why are claims-made contract more popular in long-tail lines than in short-tail lines? And
- Why are claims-made contract more popular in personal liability lines than in commercial liability lines?

The theory proposed by Doherty (1991) suggests that claims-made contract are an answer to an increase in the uncertainty of the legal environment in which insurers operate. This theory explains well why claims-made contracts should be more prevalent in long-tail lines, but it does not explain why claims-made contracts should not be more prevalent in personal liability lines. In fact, risk averse individuals should be more reluctant to assume the legal environment uncertainty than risk neutral firms. In contrast, the theory we propose based on the loss development pattern of liability claims and the difference between the discount factors of insured agents and insurance companies is able to explain the two stylized facts of short-tail versus long-tail lines and of personal versus commercial insured agent.

Another possible explanation for the prevalence of CMR over OB contracts in certain economic context, but one we did not examine in the current paper, is the uncertainty, at the time the claim is filed, as to when the loss was incurred exactly in the past. When it is hard to pinpoint the exact time a loss was incurred it becomes difficult to identify which past insurer is responsible for the claim that is filed today. It may then be optimal in terms of transaction costs to have a CMR policy rather than an OB policy. With a CMR policy, there is no uncertainty as who is financially responsible for the claim since it is the insurer underwriting the contract at the time the claim is filed. If on the other hand it is easy to pinpoint the insurer who is financially responsible, then it is less obvious that a CMR contract would be preferred. Finally, if there are solvency issues with past insurers, an insured agent could prefer to purchase a series of CMR contracts over his lifetime rather than face the possibility that, in the future, the insurer who underwrote the contract becomes bankrupt and unable to cover the loss. Using a simple Markov-switching approach with bankruptcy being an absorbing state, it is clear that the longer a loss takes to fully develop, the more likely the insurer will be bankrupt and unable to pay when the claim is finally filed. One other theory developed by Posner (1986) suggests that insurers switching to claims-made contracts were willing to continue underwriting the risk of patient injuries but did not want to assume the timing risk (i.e., when the compensation is be paid) and the corresponding inflation and investment risks. This Posner approach is somewhat linked to our approach in the sense that the switch to claims-made could be explained by a reduction in the insurers' discount factor δ_I.

The theoretical approach we used in this paper to address why and when insured agents should prefer a claim-made and reported insurance contract to an occurrence-base insurance contract allows us to draw three main conclusions. First, if agents seeking insurance are risk neutral (and therefore purchase insurance only because, for instance, they are mandated to do so by the legislation), then, only the differences in the discount rate of the insurer and of the agent will determine the purchase of such or such contract. Second, if there is some level of uncertainty as to what future losses will be, then a CMR contract can indeed help insurers get rid of some level of uncertainty as in Doherty (1991). Because a CMR insurance contract allows to price insurance using all the information that is available at the current time, neither the agent nor the insurer are locked in a contract that was signed in a past period under conditions that may no longer apply. Finally, if agents are risk averse, they may prefer the claims-made insurance contract to defer payments the first periods of their professional lives to future periods so that their income is a bit smoother than under an occurrence insurance contract.

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APPENDIX: PROOFS

Proof of Proposition 1 Because agents are risk neutral, they will purchase the contract whose cost is smaller. In other words, the CMR contract will be preferred to the OB contract if and only if $\hat{\Pi} < \overline{\Pi}$. We have

$$\overline{\Pi} = \sum_{t=0}^{T} \delta_A^t \overline{P} = \sum_{t=0}^{T} \delta_A^t \sum_{i=0}^{n} \delta_I^i \alpha_i L = \left(\sum_{t=0}^{T} \delta_A^t L \right) \left(\sum_{i=0}^{n} \delta_I^i \alpha_i \right),$$

and $\hat{\Pi}$ can be simplified into

$$\widehat{\Pi} = \sum_{t=0}^{n} \delta_A^t \sum_{i=0}^{t} \alpha_i L + \sum_{t=n+1}^{T} \delta_A^t \sum_{i=0}^{n} \alpha_i L + \sum_{t=T+1}^{T+n} \delta_A^t \sum_{i=t-T}^{n} \alpha_i L \tag{1}$$

$$= \sum_{i=0}^{n} \alpha_i \left(\sum_{t=i}^{T+i} \delta_A^t \right) L = \sum_{i=0}^{n} \alpha_i \delta_A^i \left(\sum_{t=0}^{T} \delta_A^t \right) L \tag{2}$$

$$= \left(\sum_{t=0}^{T} \delta_A^t L \right) \left(\sum_{i=0}^{n} \delta_A^i \alpha_i \right) \tag{3}$$

Hence, $\hat{\Pi} < \overline{\Pi}$ if and only if $\delta_A < \delta_I$. QED □.

Proof of Proposition 2 With a potential event in each of the first T periods of the agent's life, the sequence of OB premiums is

$$\overline{P}_t = \sum_{i=0}^{n} \alpha_i \delta_I^i E(\mathcal{L}_{t+i}),$$

that is, each period t OB premium depends on the n future expected losses. The CMR premiums take only current losses into account. However, these losses evolve with time.

$$\widehat{P}_t = \begin{cases} \sum_{i=0}^{t} \alpha_i E\left(\mathcal{L}_t\right) & \text{if } t < n \\ \sum_{i=0}^{n} \alpha_i E\left(\mathcal{L}_t\right) & \text{if } n \le t \le T \\ \sum_{i=t-T}^{n} \alpha_i E\left(\mathcal{L}_t\right) & \text{if } T+1 \le t \le T+n. \end{cases} \tag{4}$$

A risk neutral agent will choose the contract that offers the lowest expected discounted sum of premiums. We have

$$\overline{P} = \sum_{t=0}^{T} \delta_A^t \overline{P}_t = \sum_{t=0}^{T} \delta_A^t \left(\sum_{i=0}^{n} \alpha_i \delta_I^i E(\mathcal{L}_{t+i}) \right)$$
$$= \sum_{i=1}^{n} \alpha_i \sum_{t=0}^{T} \delta_A^t \delta_I^i E(\mathcal{L}_{t+i}) \tag{5}$$

and

$$\widehat{P} = \sum_{t=0}^{n} \delta_A^t \left(\sum_{i=0}^{t} \alpha_i E(\mathcal{L}_t) \right) + \sum_{t=n+1}^{T} \delta_A^t \left(\sum_{i=0}^{n} \alpha_i E(\mathcal{L}_t) \right) + \sum_{t=T+1}^{T+n} \delta_A^t \left(\sum_{i=t-T}^{n} \alpha_i E(\mathcal{L}_t) \right)$$
$$= \sum_{i=0}^{n} \alpha_i \sum_{t=0}^{T} \delta_A^{t+i} E(\mathcal{L}_{t+i}) \tag{6}$$

Then, $\overline{P} > \hat{P}$ if and only if $\delta_I > \delta_A$. QED □

Proof of Proposition 3 Suppose a potential event in period 0 with an invariant loss distribution $\mathcal{L}$ developing over $n + 1$ periods and such that $E(\mathcal{L}) = L$ regardless of the period the claim is made. Suppose there are no potential event after period 0.

With the same discount factor $\delta_A = \delta_I = \delta \in [0,1]$, the agent's intertemporal utility over periods 0 to n is $U_{OB}(\delta)$ in a OB contracts and $U_{\text{CMR}}(\delta)$ in a CMR contract, such that

$$DU(\delta) = u(Y - \alpha_0 L) - u(Y - \sum_{i=0}^{n} \alpha_i \delta^i L) + \sum_{i=1}^{n} \delta^i (u(Y - \alpha_i L) - u(Y))$$

It is easy to verify that $DU(0) = 0$ since $U_{\text{CMR}}(0) = u(Y - \alpha_0 L) = U_{OB}(0)$. We then show that $DU(1) > 0$.

$$U_{OB}(1) = u(Y - L) + nu(Y)$$

$$U_{CMR}(1) = \sum_{i=0}^{n} u(Y - \alpha_i L)$$

since $\sum_{i=0}^{n} \alpha_i = 1$. By concavity of u, we have that

$$\alpha_j u(Y - L) + \sum_{i=0, i \neq j}^{n} \alpha_i u(Y) < u(\alpha_j (Y - L) + \sum_{i=0, i \neq j}^{n} \alpha_i Y) = u(Y - \alpha_j L),$$

so that

$$\sum_{j=0}^{n} \left(\alpha_j u(Y - L) + \sum_{i=0, i \neq j}^{n} \alpha_i u(Y) \right) < \sum_{j=0}^{n} u(Y - \alpha_j L),$$

and then,

$$u(Y-L)+nu(Y)<\sum_{j=0}^{n}u(Y-\alpha_j L).$$

Hence, $DU(1) > 0$. QED □.

Proof of Proposition 4

Let us differentiate DU with respect to δ. We have

$$\frac{\partial DU}{\partial \delta}=\sum_{j=1}^{n}j\alpha_j\delta^{j-1}Lu'(Y-\sum_{i=0}^{n}\alpha_i\delta^i L)+\sum_{j=1}^{n}j\delta^{j-1}(u(Y-\alpha_j L)-u(Y))$$

$$=\sum_{j=1}^{n}j\delta^{j-1}[\alpha_j Lu'(Y-\sum_{i=0}^{n}\alpha_i\delta^i L)-(u(Y)-u(Y-\alpha_j L))]$$

The concavity of u implies that $\alpha_j Lu'(Y-\alpha_j L) > [u(Y)-u(Y-\alpha_j L)]$ (linear approximation of the difference is larger than the difference). Hence, $\frac{\partial DU}{\partial \delta}$ is positive at $\delta = 1$. For δ in [0,1], the sign is ambiguous however.

Let us denote $\hat{\delta}_j$ the value of δ for which $\sum_{i=0}^{n}\alpha_i\hat{\delta}_j^i = \alpha_j$. Then, for $\delta > \hat{\delta}_j$ we have $\sum_{i=0}^{n}\alpha_i\delta^i > \alpha_j$ and then,

$$\alpha_j Lu'(Y-\sum_{i=0}^{n}\alpha_i\delta^i L)>\alpha_j Lu'(Y-\alpha_j L)>u(Y)-u(Y-\alpha_j L).$$

Let us denote $\delta^* = \max_{j=0}^{n}\hat{\delta}_j$. We have, then, that $\alpha_j Lu'(Y-\sum_{i=0}^{n}\alpha_i\delta^i L) > \alpha_j Lu'(Y-\alpha_j L) > u(Y)-u(Y-\alpha_j L)$ for all $\delta \geq \delta^*$. Hence $\frac{\partial DU(\delta)}{\partial \delta} > 0$ for $\delta \geq \delta^*$.

QED □.

Proof of Proposition 5 Let us see what happens to DU when we slightly alter α_0.

$$\frac{\partial DU}{\partial \alpha_0}=-L\left[u'\left(Y-\alpha_0 L\right)-\delta^{T+1}u'\left(Y-\left(1-\alpha_0\right)L\right)\right]$$

$$+\left[\left(1-\delta\right)L\sum_{t=0}^{T}\delta^t u'\left(Y-\left(\alpha_0+\left(1-\alpha_0\right)\delta\right)L\right)\right]$$

$$\frac{\partial DU}{\partial \alpha_0}\times(-1/L)=\left[u'\left(Y-\alpha_0 L\right)-\delta^{T+1}u'\left(Y-\left(1-\alpha_0\right)L\right)\right]$$

$$-(1-\delta^{T+1})u'\left(Y-\left(\alpha_0+\left(1-\alpha_0\right)\delta\right)L\right),$$

because $\sum_{t=0}^{T}\delta^t = (1-\delta^{T+1})/(1-\delta)$. That is, $\frac{\partial DU}{\partial \alpha_0}$ is positive if

$$[u(Y-\alpha_0 L) - u'(Y-(\alpha_0+(1-\alpha_0)\delta)L)]$$

$$-\delta^{T+1}[u'(Y-(1-\alpha_0)L) - u'(Y-(\alpha_0+(1-\alpha_0)\delta)L)] \le 0$$

The term in the first bracket is always negative because $\alpha_0 L \le (\alpha_0 + (1-\alpha_0)\delta)L$ for $\alpha_0 < 1$ and u' is decreasing. The term in the second bracket is positive if $1-\alpha_0 \ge \alpha_0 + (1-\alpha_0)\delta$. That is, $\frac{\partial DU}{\partial \alpha_0}$ is positive if $\alpha_0 \le \frac{1-\delta}{2-\delta}$. Note that the fraction $\frac{1-\delta}{2-\delta}$ is decreasing in δ, with maximum value 1/2 when $\delta = 0$ and minimum value 0 when $\delta = 1$.

For any α_0 greater than $\frac{1-\delta}{2-\delta}$, the derivative $\frac{\partial DU}{\partial \alpha_0}$ can be positive or negative. Let us find the limit value of this derivative as $\alpha_0 \to 1$. We have

$$\frac{\partial DU}{\partial \alpha_0}\Big|_{\alpha_0 \to 1} = -L\left[u'(Y-L) - \delta^{T+1}u'(Y)\right] + (1-\delta)L\sum_{t=0}^{T}\delta^t u'(Y-L)$$

$$= L\left[\delta^{T+1}u'(Y) - u'(Y-L)\left(1-\sum_{t=0}^{T}\delta^t + \delta\sum_{t=0}^{T}\delta^t\right)\right]$$

$$= \delta^{T+1}\left[u'(Y) - u'(Y-L)\right]L < 0$$

For high values of α_0, the CMR advantage becomes lower as the tail becomes even thinner. QED □.

Notes

1. The following professional protections are offered on a claims-made basis: Directors and officers liability, Private company liability, Employment practices liability, Fiduciary liability, Bankers professional liability, Insurance company liability, Security & privacy liability, and Employed lawyers professional liability.